

Name: _____ Date: _____

Instructions. Show all work with clear, logical steps. No work or hard-to-follow work will lose points. Scientific calculators are allowed. **2** points for name/date

Problem 1. (2 points each) Given

$$f_x(x, y) = 8xy + \frac{6}{x}, \quad f_y(x, y) = 4x^2 + \frac{6}{y},$$

find f_{xx} , f_{yy} , and f_{xy} .

Solution:

Differentiate f_x with respect to x to get f_{xx} :

$$f_{xx}(x, y) = \frac{\partial}{\partial x} \left(8xy + \frac{6}{x} \right) = 8y - \frac{6}{x^2}.$$

Differentiate f_y with respect to y to get f_{yy} :

$$f_{yy}(x, y) = \frac{\partial}{\partial y} \left(4x^2 + \frac{6}{y} \right) = -\frac{6}{y^2}.$$

Differentiate f_x with respect to y (or f_y with respect to x) to get the mixed partial:

$$f_{xy}(x, y) = \frac{\partial}{\partial y} \left(8xy + \frac{6}{x} \right) = 8x.$$

(As a check, $\frac{\partial}{\partial x}(4x^2 + \frac{6}{y}) = 8x$ as well, so $f_{xy} = f_{yx} = 8x$.)

Answer: $f_{xx} = 8y - \frac{6}{x^2}$, $f_{yy} = -\frac{6}{y^2}$, $f_{xy} = 8x$.

Problem 2. (2 points) If a function $z = f(x, y)$ satisfies

$$f_{xy} = 13x^2 \sin(xy)e^{x+y},$$

What is f_{yx} ?

Solution:

If the second partial derivatives are continuous (Clairaut's theorem / equality of mixed partials), then

$$f_{yx}(x, y) = f_{xy}(x, y) = 13x^2 \sin(xy)e^{x+y}.$$

Answer: $f_{yx} = 13x^2 \sin(xy)e^{x+y}$.