

Name: _____ Date: _____

Instructions. Show all work with clear, logical steps. Scientific calculators are allowed.

Problem 1. (2 points each) If the given geometric series converges, find its **sum**. If not, state that it **diverges**.

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$$\sum_{n=0}^{\infty} \left(\frac{3}{2}\right)^n$$

Solution: The common ratio is $r = \frac{3}{2}$. Since $|r| = 1.5 > 1$, the series **diverges**.

Diverges.

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$$\sum_{n=0}^{\infty} 6 \left(\frac{1}{9}\right)^n$$

Solution: Here $a = 6$ and $r = \frac{1}{9}$. Since $|r| < 1$, the series converges. The sum is

$$S = \frac{a}{1-r} = \frac{6}{1-\frac{1}{9}} = \frac{6}{\frac{8}{9}} = \frac{6 \times 9}{8} = \frac{54}{8} = \boxed{\frac{27}{4}}.$$

Problem 2. (4 points) Express

$$\frac{3}{1-2x}$$

as a power series. Determine the radius of convergence.

Solution: Recall the geometric series formula

$$\frac{1}{1-r} = \sum_{n=0}^{\infty} r^n, \quad \text{for } |r| < 1.$$

Here $r = 2x$, so

$$\frac{3}{1-2x} = 3 \sum_{n=0}^{\infty} (2x)^n = \sum_{n=0}^{\infty} 3(2^n)x^n.$$

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Radius of convergence: The series converges when $|2x| < 1 \Rightarrow |x| < \frac{1}{2}$.

$$R = \frac{1}{2}$$