Name: _____   Date: _____

Instructions. Show all work with clear, logical steps. No work or hard-to-follow work will lose points. Scientific calculators are allowed. **2** points for name/date

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Problem 1. (8 points, 2 points each) Given the information in the table below, use the discriminant D to find and classify any critical points for the function $g(x, y)$. (**Hint:** there are four critical points in the table).

(x_0, y_0)	$g(x_0, y_0)$	$g_x(x_0, y_0)$	$g_y(x_0, y_0)$	$g_{xx}(x_0, y_0)$	$g_{xy}(x_0, y_0)$	$g_{yy}(x_0, y_0)$
$(0, 1)$	0	3	0	0	-2	4
$(4, 3)$	-3	0	0	-1	2	-6
$(2, 7)$	15	0	0	4	5	8
$(5, 6)$	4	0	0	3	5	2
$(-2, 8)$	2	0	0	2	2	2

1)

8

● ●**[?pts]** Given the information in the table below, find and classify any critical points for the function $g(x,y)$.

(x_0, y_0)	$g(x_0, y_0)$	$g_x(x_0, y_0)$	$g_y(x_0, y_0)$	$g_{xx}(x_0, y_0)$	$g_{xy}(x_0, y_0)$	$g_{yy}(x_0, y_0)$
$(0,1)$	0	3	0	0	-2	4
$(4,3)$	-3	0	0	-1	2	-6
$(2,7)$	15	0	0	4	5	8
$(5,6)$	4	0	0	3	5	2
$(-2,8)$	2	0	0	2	2	2

Solution: First check for each point that both g_x and g_y are 0.

- Hence $(0,1)$ is not a critical point. **[1 pt]**

Next, let's compute the discriminant of each point.

- $\underline{(4,3)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = -1 \cdot (-6) - (2)^2 = +2$

- $\underline{(2,7)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = 4 \cdot 8 - (5)^2 = 7$

- $\underline{(5,6)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = 3 \cdot 2 - (5)^2 = -19$

- $\underline{(-2,8)}$: $D = g_{xx}g_{yy} - (g_{xy})^2 = 2 \cdot 2 - (2)^2 = 0$

When $D > 0$, we have a relative extrema. Hence $(4,3)$ and $(2,7)$ are relative extrema. To determine whether they are maxs or mins, we need to check the sign of g_{xx}.

- $\underline{(4,3)}$: $g_{xx} = -1 < 0$. Hence $(4,3)$ is a relative max. **[2 pt]**

- $\underline{(2,7)}$: $g_{xx} = 4 > 0$. Hence $(2,7)$ is a relative min. **[1 pt]**

When $D < 0$, we have a saddle point. Hence $(5,6)$ is a saddle point. **[2 pt]**

When $D = 0$, the test is inconclusive. Hence at $(-2,8)$ the test is inconclusive. **[1 pt]**