MA 16020 – EXAM FORMULAS

THE SECOND DERIVATIVE TEST

Suppose f is a function of two variables x and y, and that all the second-order partial derivatives are continuous. Let

$$d = f_{xx}f_{yy} - (f_{xy})^2$$

and suppose (a, b) is a critical point of f.

1. 1.If $d(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a relative minimum at (a, b).
2. 2.If $d(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a relative maximum at (a, b).
3. 3.If $d(a, b) < 0$, then f has a saddle point at (a, b).
4. 4.If $d(a, b) = 0$, the test is inconclusive.

LAGRANGE EQUATIONS

For the function $f(x, y)$ subject to the constraint $g(x, y) = c$, the Lagrange equations are

$$f_x = \lambda g_x \qquad f_y = \lambda g_y \qquad g(x, y) = c$$

GEOMETRIC SERIES

If $0 < |r| < 1$, then

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

TAYLOR SERIES

The Taylor series of $f(x)$ about $x = c$ is the power series

$$\sum_{n=0}^{\infty} a_n(x-c)^n \quad \text{where} \quad a_n = \frac{f^{(n)}(c)}{n!}$$

Examples:

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } -\infty < x < \infty; \qquad \ln x = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(x-1)^n, \text{ for } 0 < x \le 2$$

VOLUME & SURFACE AREA

Right Circular Cylinder

$V = \pi r^2 h$

$$SA = \begin{cases} 2\pi r^2 + 2\pi r h \\ \pi r^2 + 2\pi r h \end{cases}$$

Sphere

$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

Right Circular Cone

$V = \frac{1}{3}\pi r^2 h$

$SA = \pi r\sqrt{r^2 + h^2} + \pi r^2$

Practice Problems

1. Evaluate $\int \frac{1}{(3x-1)^4} dx$.

   A. $-\frac{12}{(3x-1)^5} + C$

   B. $-\frac{1}{9(3x-1)^3} + C$

   C. $\frac{1}{(3x-1)^3} + C$

   D. $-\frac{1}{3(3x-1)^3} + C$

   E. $-\frac{4}{(3x-1)^5} + C$

2. Evaluate $\int e^{3-2x} dx$.

   A. $-2e^{3-2x} + C$

   B. $-\frac{1}{2}e^{3-2x} + C$

   C. $\frac{e^{4-2x}}{4-2x} + C$

   D. $\frac{1}{3}e^{3-2x} + C$

   E. $\frac{e^{3-2x}}{3-2x} + C$

3. Find a function f whose tangent line has slope $x\sqrt{5-x^2}$ for each value of x and whose graph passes through the point (2,10).

   A. $f(x) = -\frac{1}{3}(5-x^2)^{3/2}$

   B. $f(x) = \frac{2}{3}(5-x^2)^{3/2} + \frac{28}{3}$

   C. $f(x) = \frac{1}{3}(5-x^2)^{3/2} + \frac{29}{3}$

   D. $f(x) = -\frac{1}{3}(5-x^2)^{3/2} + \frac{31}{3}$

   E. $f(x) = \frac{3}{2}(5-x^2)^{3/2} + \frac{17}{2}$

4. Evaluate $\int x \ln(x^2) dx$.

   A. $\frac{1}{2}x^2 \ln x^2 - \frac{1}{2}x^2 + C$

   B. $\frac{1}{2}x^2 \ln x^2 - \frac{1}{2}x + C$

   C. $\frac{1}{2}x^2 \ln x^2 - \frac{1}{6}x^3 + C$

   D. $x \ln x^2 + \frac{1}{x} + C$

   E. $\frac{1}{2}x^2 \ln x - \frac{1}{2}x^2 + C$

5. The area of the region bounded by the curves $y = x^2 + 1$ and $y = 3x + 5$ is

A. $\frac{125}{6}$
B. $\frac{56}{3}$
C. $\frac{27}{2}$
D. $\frac{25}{6}$
E. $\frac{32}{3}$

6. If $f(x, y) = (xy + 1)^2 - \sqrt{y^2 - x^2}$, evaluate $f(-2, 1)$.

A. 1
B. $1 - \sqrt{5}$
C. Not defined
D. $-1 - \sqrt{5}$
E. $-1 - \sqrt{3}$

7. A paint store carries two brands of latex paint. Sales figures indicate that if the first brand is sold for x_1 dollars per gallon and the second for x_2 dollars per gallon, the demand for the first brand will be $D_1(x_1, x_2) = 100 + 5x_1 - 10x_2$ gallons per month and the demand for the second brand will be $D_2(x_1, x_2) = 200 - 10x_1 + 15x_2$ gallons per month. Express the paint store's total monthly revenue, R, as a function of x_1 and x_2.

A. $R = x_1 D_1(x_1, x_2) + x_2 D_2(x_1, x_2)$
B. $R = D_1(x_1, x_2) + D_2(x_1, x_2)$
C. $R = D_1(x_1, x_2) D_2(x_1, x_2)$
D. $R = x_2 D(x_1, x_2) + x_1 D_2(x_1, x_2)$
E. $R = x_1 x_2 + D_1(x_1, x_2) D_2(x_1, x_2)$

8. Compute $\dfrac{\partial z}{\partial x}$, where $z = \ln(xy)$.

A. $\dfrac{1}{x}$
B. $\dfrac{1}{y}$
C. $\dfrac{1}{xy}$
D. $\dfrac{1}{x} + \dfrac{1}{y}$
E. $\dfrac{y}{x}$

9. Compute f_{uv} if $f = uv + e^{u+2v}$.

A. 0
B. $u + 2e^{u+2v}$
C. $v + 2e^{u+2v}$
D. $1 + 2e^{u+2v}$
E. $1 + e^{u+2v}$

10. Find and classify the critical points of $f(x,y) = (x-2)^2 + 2y^3 - 6y^2 - 18y + 7$.

A. (2,3) saddle point; (2,−1) relative minimum

B. (2,3) relative maximum; (2,−1) relative minimum

C. (2,3) relative minimum; (2,−1) relative maximum

D. (2,3) relative maximum; (2,−1) saddle point

E. (2,3) relative minimum; (2,−1) saddle point

11. A manufacturer sells two brands of foot powder, brand A and brand B. When the price of A is x cents per can and the price of B is y cents per can the manufacturer sells $40-8x+5y$ thousand cans of A and $50+9x-7y$ thousand cans of B. The cost to produce A is 10 cents per can and the cost to produce B is 20 cents per can. Determine the selling price of brand A which will maximize the profit.

A. 40 cents

B. 45 cents

C. 15 cents

D. 50 cents

E. 35 cents

12. Use increments to estimate the change in z at (1,3) if $\frac{\partial z}{\partial x} = 2x - 4$, $\frac{\partial z}{\partial y} = 2y + 7$, the change in x is 0.3 and the change in y is 0.5.

A. 7.1

B. 2.9

C. 4.9

D. 5.9

E. 6.3

13. Using x worker-hours of skilled labor and y worker-hours of unskilled labor, a manufacturer can produce $f(x,y) = x^2y$ units. Currently 16 worker-hours of skilled labor and 32 worker-hours of unskilled labor are used. If the manufacturer increases the unskilled labor by 10 worker-hours, use calculus to estimate the corresponding change that the manufacturer should make in the level of skilled labor so that the total output will remain the same.

A. Reduce by 4 hours.

B. Reduce by 10 hours.

C. Reduce by $\frac{5}{4}$ hours.

D. Reduce by $\frac{5}{2}$ hours.

E. Reduce by 5 hours.

14. Find the maximum value of the function $f(x, y) = 20x^{3/2}y$ subject to the constraint $x + y = 60$. Round your answer to the nearest integer.

A. 84,654

B. 188,334

C. 4,320

D. 259,200

E. 103,680

15. Evaluate $\int_1^2 \int_0^1 (2x + y)\, dydx$.

A. $\frac{9}{2}$

B. $\frac{5}{2}$

C. $\frac{3}{2}$

D. $\frac{7}{2}$

E. $\frac{1}{2}$

16. The general solution of the differential equation $\frac{dy}{dx} = 2y + 1$ is:

A. $x = y^2 + y + C$

B. $2y + 1 = Ce^{2x}$

C. $y = 2xy + x + C$

D. $y = Ce^{2x} - 2y - 1$

E. $y = Ce^{2x}$

17. The value, V, of a certain \$1500 IRA account grows at a rate equal to 13.5% of its value. Its value after t years is:

A. $V = 1500e^{-0.135t}$

B. $V = 1500 + 0.135t$

C. $V = 1500e^{0.135t}$

D. $V = 1500(1 + 0.135t)$

E. $V = 1500 \ln(0.135t)$

18. It is estimated that t years from now the population of a certain town will be increasing at a rate of $5 + 3t^{2/3}$ hundred people per year. If the population is presently 100,000, by how many people will the population increase over the next 8 years?

A. 100

B. 9,760

C. 6,260

D. 24,760

E. 17,260

19. Calculate the improper integral $\int_0^\infty xe^{-x^2}\,dx$.

A. $-\frac{1}{2}$

B. 1

C. $\frac{1}{2}$

D. $\frac{5}{2}$

E. The integral diverges.

20. An object moves so that its velocity after t minutes is given by the formula $v = 20e^{-0.01t}$. The distance it travels during the 10th minute is

A. $\int_0^{10} 20e^{-0.01t} dt$

B. $\int_9^{10} (-20e^{-0.01t}) dt$

C. $\int_0^{10} (-20e^{-0.01t}) dt$

D. $\int_9^{10} 20e^{-0.01t} dt$

E. $\int_9^{10} (-0.2e^{-0.01t}) dt$

21. Find the sum of the series $\sum_{n=1}^{\infty} \left(-\frac{2}{3}\right)^n$.

A. $\frac{2}{5}$

B. $-\frac{2}{5}$

C. $\frac{3}{2}$

D. $-\frac{3}{2}$

E. The series diverges.

22. Use a Taylor polynomial of degree 2 to approximate $\int_0^{0.1} \frac{100}{x^2+1}\,dx$. Round your answer to five decimal places.

A. 9.96687

B. 10.00000

C. 9.96677

D. 9.66667

E. 9.96667

23. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{n3^n x^n}{5^{n+1}}$.

A. $\frac{5}{3}$

B. 1

C. $\frac{3}{25}$

D. $\frac{3}{5}$

E. ∞

24. Find the Taylor series of $f(x) = \dfrac{x}{2+x^2}$ at $x = 0$.

A. $\sum_{n=0}^{\infty} \dfrac{x^{n+1}}{2^{n+1}}$

B. $\sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{2^n}$

C. $\sum_{n=0}^{\infty} (-1)^n 2^{n-1} x^{2n+1}$

D. $\sum_{n=0}^{\infty} \dfrac{x^{2n}}{2^{n-1}}$

E. $\sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n+1}}{2^{n+1}}$

25. Write the following infinite series in summation notation.

$$5 - \frac{7}{8} + \frac{9}{27} - \frac{11}{64} + \dots$$

A. $\sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{2n+5}{n^3}$

B. $\sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{2n+3}{n^3}$

C. $\sum_{n=1}^{\infty} (-1)^{n+1} \dfrac{3n+2}{n^3}$

D. $\sum_{n=1}^{\infty} (-1)^{n} \dfrac{2n+5}{2^n}$

E. $\sum_{n=1}^{\infty} (-1)^{n} \dfrac{2n+3}{2^n}$

26. Determine which of the following series converge.

I. $\sum_{k=2}^{\infty} \frac{k^2}{5^k}$

II. $\sum_{k=3}^{\infty} \frac{(3k+1)\pi^{2k}}{10^{k+1}}$

III. $\sum_{k=1}^{\infty} \frac{k!}{(-2)^k}$

A. III

B. I & II

C. I & III

D. II & III

E. II

27. Find the Taylor series about $x = 0$ for the indefinite integral

$$\int xe^{-x^3}\,dx.$$

A. $\sum_{n=0}^{\infty} \frac{1}{n!(3n+1)} x^{3n+2} + C$

B. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(3n+2)} x^{3n+2} + C$

C. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(3n+1)} x^{3n+2} + C$

D. $\sum_{n=0}^{\infty} \frac{1}{n!(3n+2)} x^{3n+2} + C$

E. $\sum_{n=0}^{\infty} \frac{(-1)^n}{n!(3n+1)} x^{3n+1} + C$

28. A patient is given an injection of 50 milligrams of a drug every 24 hours. After t days, the fraction of the drug remaining in the patient's body is

$$f(t) = 2^{-t/3}.$$

If the treatment is continued indefinitely, approximately how many milligrams of the drug will eventually be in the patient's body just prior to an injection?

A. 202.7

B. 152.7

C. 305.4

D. 242.4

E. 192.4

29. Compute $\int (\sin x - \cos x)(\sin x + \cos x)^5 \, dx$.

A. $\frac{1}{6}(-\cos x + \sin x)^6 + C$
B. $-6(-\cos x + \sin x)^6 + C$
C. $-\frac{1}{6}(\sin x + \cos x)^6 + C$
D. $6(\sin x + \cos x)^6 + C$
E. $\frac{1}{6}(\sin x + \cos x)^6 + C$

30. Evaluate $\int x^2 \cos(-5x) \, dx$.

A. $-\frac{1}{5}x^2 \sin(-5x) + \frac{2}{25}x \cos(-5x) + \frac{2}{125} \sin(-5x) + C$
B. $\frac{1}{5}x^2 \sin(-5x) - \frac{2}{25}x \cos(-5x) - \frac{2}{125} \sin(-5x) + C$
C. $-5x^2 \sin(-5x) + 50x \cos(-5x) + 250 \sin(-5x) + C$
D. $5x^2 \cos(-5x) - 50x \sin(-5x) - 250 \cos(-5x) + C$
E. $5x^2 \sin(-5x) - 50x \cos(-5x) - 250 \sin(-5x) + C$

31. Evaluate $\int_e^5 \frac{\ln(x^4)}{x} \, dx$.

A. $\frac{1}{8}(25 - e^2)$
B. $2(25 - e^2)$
C. $2(\ln 5)^2 - 2$
D. $\frac{1}{8}(\ln 5)^2 - \frac{1}{8}$
E. $\ln(25) - 2$

32. Find the volume of the solid generated by revolving the region bounded by:

$$y = 3e^{2x}, y = 0, x = 1, \text{ and } x = 3$$

about the x-axis.

A. $\frac{3\pi}{4}(e^8 - 1)e^4$
B. $\frac{3\pi}{4}(e^8 - 1)e^2$
C. $\frac{9\pi}{2}(e^4 - 1)e^2$
D. $\frac{9\pi}{4}(e^8 - 1)e^4$
E. $\frac{3\pi}{2}(e^4 - 1)e^2$

33. Find the volume of the solid which has square cross-sections with side length $5x^2$ at each point $2 \le x \le 4$.

A. $\int_2^4 5\pi x^2\, dx$

B. $\int_2^4 5x^2\, dx$

C. $\int_2^4 5x^4\, dx$

D. $\int_2^4 25\pi x^4\, dx$

E. $\int_2^4 25x^4\, dx$

34. The velocity of a car over the time period $0 \le t \le 3$ is given by the function

$$v(t) = 60te^{\frac{-t}{4}}$$

miles per hour, where t is time in **hours**. What was the distance the car traveled in the first 90 **minutes**? Round your answer to two decimal places.

A. 166.42 miles

B. 156.19 miles

C. 126.63 miles

D. 75.85 miles

E. 52.78 miles

35. Given that $f(x, y) = \tan(xy^3)$, compute $f_x(2\pi, \frac{1}{2})$.

A. $\frac{3}{2}$

B. $\frac{\pi}{2}$

C. 1

D. 6π

E. $\frac{1}{4}$

36. Let $h(x, y) = y\sin(xy)$. Find $\frac{\partial^2 h}{\partial y \partial x}$.

A. $-2xy\sin(xy)$

B. $2y\cos(xy) - xy^2\sin(xy)$

C. $-y^3\sin(xy)$

D. $\cos(xy) + y^2\sin(xy)$

E. $(x + 1)\cos(xy) - x^2y\sin(xy)$

37. A nature preserve wishes to construct a large compound which will hold both lions and gazelles. They currently have 6 gazelles. They estimate that if they use an area of A square miles and introduce L lions, then they will be able to support a population of G gazelles, given by the function

$$G(A, L) = 6 + 40A - A^2 - 18L^2 + 176L - 8AL$$

What conditions will lead to the largest number of gazelles?

A. $L = 3,\ A = 5$

B. $L = 4,\ A = 4$

C. $L = 5,\ A = 4$

D. $L = 5,\ A = 3$

E. There are no such conditions because the function does not have a maximum.

38. Evaluate $\iint_R (e^{x^2+1})\, dA$, where R is the region indicated by the boundaries below:

$$0 \le x \le 1; \quad 0 \le y \le x$$

A. 0

B. $\frac{1}{2}e$

C. $\frac{1}{2}e^2$

D. $\frac{1}{2}(e^2 - e)$

E. $e^2 - e$

39. Compute AB and BA, if possible, for the matrices:

$$A = \begin{bmatrix} 2 & -1 \\ 0 & -3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 \\ -5 & 1 \\ 2 & 0 \end{bmatrix}$$

A. BA is not possible, and $AB = \begin{bmatrix} -1 & -3 \\ -11 & -3 \\ 4 & 0 \end{bmatrix}$

B. BA is not possible, and $AB = \begin{bmatrix} -1 & -11 & 4 \\ -3 & -3 & 0 \end{bmatrix}$

C. AB is not possible, and $BA = \begin{bmatrix} 0 & -3 \\ -10 & 2 \\ 4 & -2 \end{bmatrix}$

D. AB is not possible, and $BA = \begin{bmatrix} 0 & -10 & 4 \\ -3 & 2 & -2 \end{bmatrix}$

E. Both AB and BA are not possible.

40. Find the general solution to the differential equation

$$-x^5 \sin x + xy' = 3y, \qquad x > 0$$

A. $y = -x \cos x - \sin x + C$

B. $y = -x \cos x + \sin x + C$

C. $y = x \cos x + \sin x + C$

D. $y = -x^4 \cos x + x^3 \sin x + Cx^3$

E. $y = x^4 \cos x + x^3 \sin x + Cx^3$

41. The amount of carbon, in grams, in a sample of soil is given by a function, $F(t)$, satisfying the differential equation:

$$F' + aF - b = 0$$

where a and b are constants, and time, t, is measured in years. If the sample originally contains 10 grams of carbon, which expression represents the amount of carbon present after 5 years?

A. $\frac{b}{a} + (10 - \frac{b}{a})e^{5a}$

B. $\frac{b}{a} + (10 - \frac{b}{a})e^{-5a}$

C. $ab + (10 - ab)e^{-5a}$

D. $ab + (10 - ab)e^{5a}$

E. $\frac{b}{a} + 10e^{5a}$

42. Let $M = \begin{matrix} 4 & 3 \\ -2 & -1 \end{matrix}$. Compute $3M - M^2$.

A. $\begin{matrix} 2 & 0 \\ 0 & 2 \end{matrix}$

B. $\begin{matrix} 2 & 1 \\ 0 & 2 \end{matrix}$

C. $\begin{matrix} 2 & 0 \\ -1 & 2 \end{matrix}$

D. $\begin{matrix} 2 & 1 \\ 1 & 2 \end{matrix}$

E. $\begin{matrix} 2 & -1 \\ 1 & 2 \end{matrix}$

Answers to Practice Problems

1. B	2. B	3. D	4. A
5. A	6. C	7. A	8. A
9. D	10. E	11. A	12. D
13. D	14. E	15. D	16. B
17. C	18. B	19. C	20. D
21. B	22. E	23. A	24. E
25. B	26. B	27. B	28. E
29. C	30. A	31. C	32. D
33. E	34. E	35. E	36. B
37. B	38. D	39. C	40. D
41. B	42. A	43. A	44. B
45. B	46. A	47. D	48. D