MA 16020 – Applied Calculus II: Lecture 8
Integration by Parts II

Integration by Parts Formula

Remark

This is the **Integration by Parts formula***:*

$$\int u\,dv = uv - \int v\,du$$

- u – function we **differentiate** (derivative should be simpler)
- dv – function we **integrate** (integral should be easy)

Choosing u – LATE Rule

LATE: guides which function to differentiate

1. L: Logarithmic
2. A: Algebraic
3. T: Trigonometric
4. E: Exponential

- Pick $u =$ the first function appearing in LATE
- Pick $dv =$ the remaining function

Example 1 – Definite Integrals

Apply integration by parts to the following definite integrals:

1. $\displaystyle\int_1^e x \ln \sqrt[3]{x}\, dx$
2. $\displaystyle\int_0^1 (x^2+1)e^{-x}\, dx$
3. Evaluate:
$$\int_0^2 3xe^{2x}\, dx$$
    - Hint: Pick u using LATE.
    - Evaluate $uv - \int v\, du$ at the bounds.
    - Focus on simplifying the algebra and trigonometry.

Solution to (1): $\int_1^e x \ln \sqrt[3]{x}\, dx$

We simplify first:

$$\ln \sqrt[3]{x} = \ln(x^{1/3}) = \frac{1}{3} \ln x$$

So the integral becomes:

$$\int_1^e x \cdot \frac{1}{3} \ln x\, dx = \frac{1}{3} \int_1^e x \ln x\, dx$$

Use integration by parts:

$$u = \ln x \qquad dv = x\, dx$$

$$du = \frac{1}{x}\, dx \qquad v = \frac{x^2}{2}$$

Solution to (1)

Apply the formula:

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \cdot \frac{1}{x} \, dx = \frac{x^2}{2} \ln x - \int \frac{x}{2} \, dx$$

So,

$$\int x \ln x \, dx = \frac{x^2}{2} \ln x - \frac{x^2}{4}$$

Now evaluate from 1 to e:

$$\frac{1}{3} \left[\left(\frac{e^2}{2} \ln e - \frac{e^2}{4} \right) - \left(\frac{1}{2} \ln 1 - \frac{1}{4} \right) \right]$$

$$= \frac{1}{3} \left[\left(\frac{e^2}{2} - \frac{e^2}{4} \right) - \left(0 - \frac{1}{4} \right) \right] = \frac{1}{3} \left(\frac{e^2}{4} + \frac{1}{4} \right)$$

$$\boxed{= \frac{1}{12}(e^2 + 1)}$$

Solution to (2): $\int_0^1 (x^2+1)e^{-x}\,dx$

Let's apply integration by parts. Pick:

$$u = x^2 + 1 \qquad dv = e^{-x}\,dx$$

$$du = 2x\,dx \qquad v = -e^{-x}$$

Use the formula:

$$\int u\,dv = uv - \int v\,du$$

$$= -(x^2+1)e^{-x}\Big|_0^1 + \int_0^1 2xe^{-x}\,dx$$

Now evaluate the first term:

$$-(x^2+1)e^{-x}\Big|_0^1 = -[(1^2+1)e^{-1} - (0^2+1)e^0] = -[2e^{-1}-1] = 1 - \frac{2}{e}$$

Now solve the second integral using integration by parts again:

$$\int_0^1 2xe^{-x}\,dx$$

Solution to (2)

Let:

$$u = 2x \qquad dv = e^{-x}dx$$

$$du = 2\,dx \qquad v = -e^{-x}$$

Then:

$$= -2xe^{-x}\Big|_0^1 + \int_0^1 2e^{-x}\,dx = -2e^{-1} + \int_0^1 2e^{-x}\,dx$$

$$= -\frac{2}{e} + 2(1 - \frac{1}{e}) = 2 - \frac{4}{e}$$

Putting it all together:

$$\int_0^1 (x^2+1)e^{-x}\,dx = \left(1 - \frac{2}{e}\right) + \left(2 - \frac{4}{e}\right) = \boxed{3 - \frac{6}{e}}$$