MA 16020 – Applied Calculus II: Lecture 7
Integration by Parts

Motivation

- Some integrals of products are hard to compute directly, e.g.,

$$\int xe^x \, dx$$

- Idea: **transfer the derivative from one function to another** to simplify
- Comes from the **product rule**:

$$\frac{d}{dx}[uv] = u'v + uv' \implies \int u \, dv = uv - \int v \, du$$

Integration by Parts Formula

Remark

This is the **Integration by Parts formula***:*

$$\int u\,dv = uv - \int v\,du$$

- u – function we **differentiate** (derivative should be simpler)
- dv – function we **integrate** (integral should be easy)

Choosing u – LATE Rule

LATE: guides which function to differentiate

1. L: Logarithmic
2. A: Algebraic
3. T: Trigonometric
4. E: Exponential

- Pick u = the first function appearing in LATE
- Pick dv = the remaining function

Example 1 – Practice Integrals

Apply integration by parts to the following integrals. Try to identify u and dv using LATE.

1. $\displaystyle\int xe^{2x}\,dx$
2. $\displaystyle\int \ln x\,dx$
3. $\displaystyle\int 2x\cos(3x)\,dx$

- Hint: Choose u from the first function in LATE.
- dv is the remaining part to integrate.

Solution to (1): $\int xe^{2x}\,dx$

Use integration by parts:

$$u = x \qquad dv = e^{2x}\,dx$$

$$du = dx \qquad v = \frac{1}{2}e^{2x}$$

Apply the formula:

$$\int xe^{2x}\,dx = uv - \int v\,du = x \cdot \frac{1}{2}e^{2x} - \int \frac{1}{2}e^{2x}\,dx$$

$$= \frac{x}{2}e^{2x} - \frac{1}{4}e^{2x} + C$$

$$\boxed{= \frac{e^{2x}}{4}(2x-1) + C}$$

Solution to (2): $\int \ln x \, dx$

Use integration by parts with:

$$u = \ln x \qquad dv = dx$$
$$du = \frac{1}{x} dx \qquad v = x$$

Apply the formula:

$$\int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - \int 1 \, dx$$

$$= x \ln x - x + C$$

$$\boxed{= x(\ln x - 1) + C}$$

Solution to (3): $\int 2x\cos(3x)\,dx$

Use integration by parts:

$$u = 2x \qquad dv = \cos(3x)\,dx$$
$$du = 2\,dx \qquad v = \frac{1}{3}\sin(3x)$$

Apply the formula:

$$\int 2x\cos(3x)\,dx = 2x\cdot\frac{1}{3}\sin(3x) - \int \frac{1}{3}\cdot 2\sin(3x)\,dx$$

$$= \frac{2x}{3}\sin(3x) - \frac{2}{3}\int \sin(3x)\,dx = \frac{2x}{3}\sin(3x) + \frac{2}{9}\cos(3x) + C$$

$$\boxed{= \frac{2x}{3}\sin(3x) + \frac{2}{9}\cos(3x) + C}$$

Example 2 Problem

Find the function $f(x)$ whose slope at any given x-value is

$$f'(x) = (x - 5)e^{-4x},$$

and whose graph passes through the point $(5, 0)$.

Solution

We need to integrate:

$$f'(x) = (x-5)e^{-4x}.$$

Use integration by parts with $u = x - 5$, $dv = e^{-4x}\,dx$.

$$du = dx, \quad v = \int e^{-4x} dx = -\tfrac{1}{4}e^{-4x}.$$

So,

$$f(x) = uv - \int v\,du = (x-5)\Big(-\tfrac{1}{4}e^{-4x}\Big) - \int \Big(-\tfrac{1}{4}e^{-4x}\Big)dx.$$

$$f(x) = -\tfrac{1}{4}(x-5)e^{-4x} + \tfrac{1}{16}e^{-4x} + C.$$

Use the condition $f(5) = 0$:

$$f(5) = -\tfrac{1}{4}(0)\cdot e^{-20} + \tfrac{1}{16}e^{-20} + C = 0,$$

$$C = -\tfrac{1}{16}e^{-20}.$$