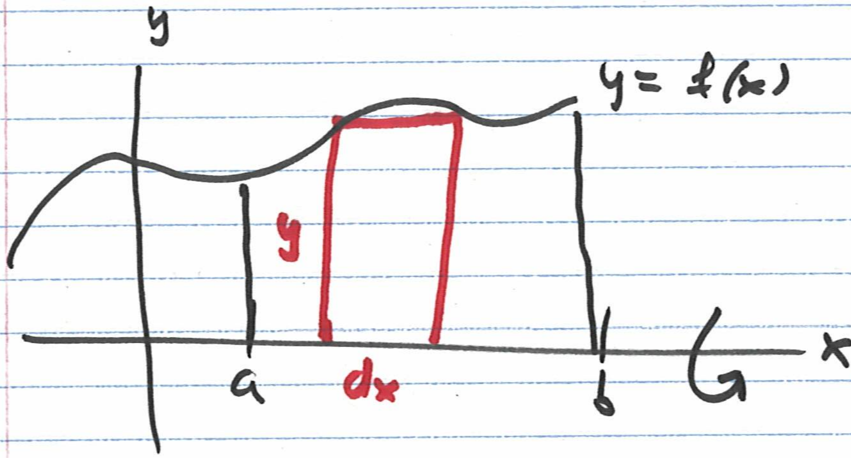
Lesson 16 . Solids of Revolution

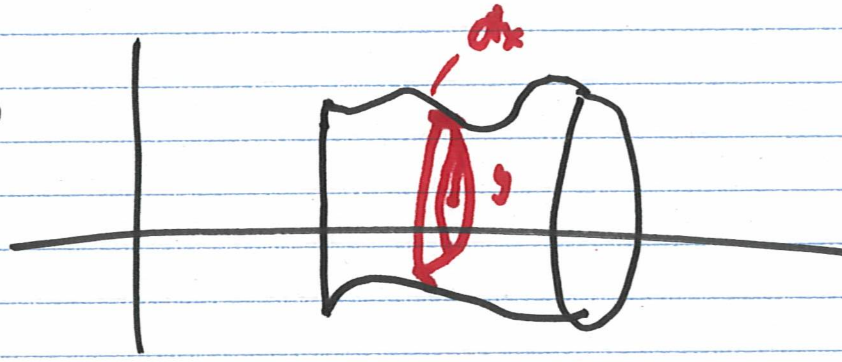
Washers / Disks II :

Method	Slice Shape	Slice Orientation
Disk / Washer	Cross Section $\perp$ axis of rotation	perpendicular Slices

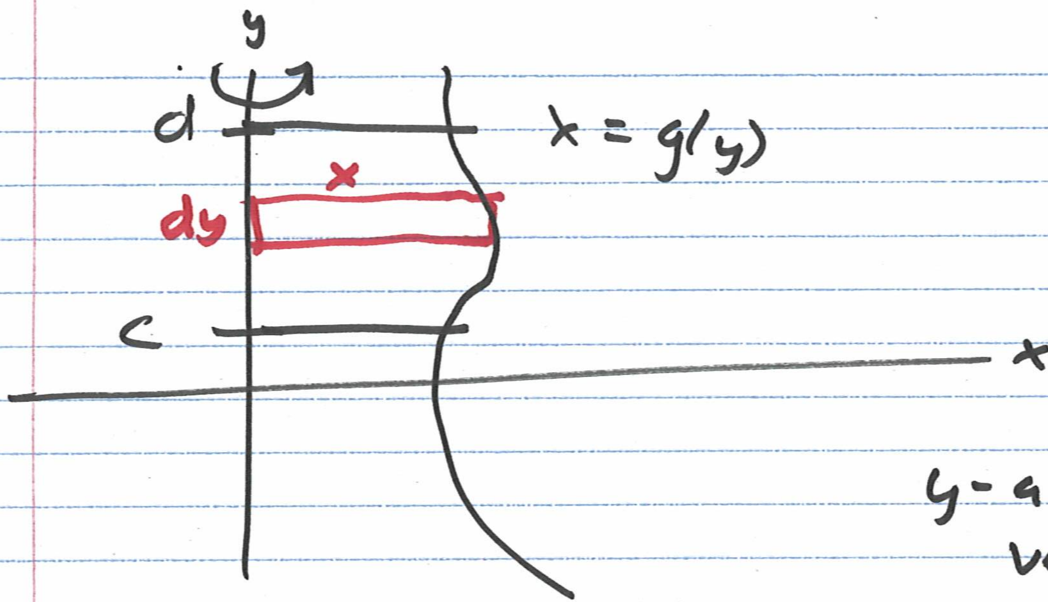


x - axis (y = 0)
- horizontal axis

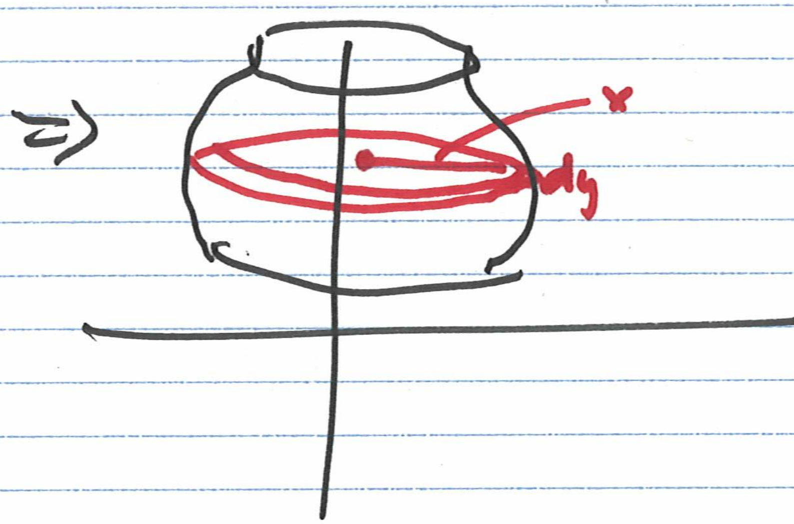
||
vertical disk



disk method $$V = \pi \int_a^b \left(f(x) \right)^2 dx$$



y-axis: x = 0
vertical axis



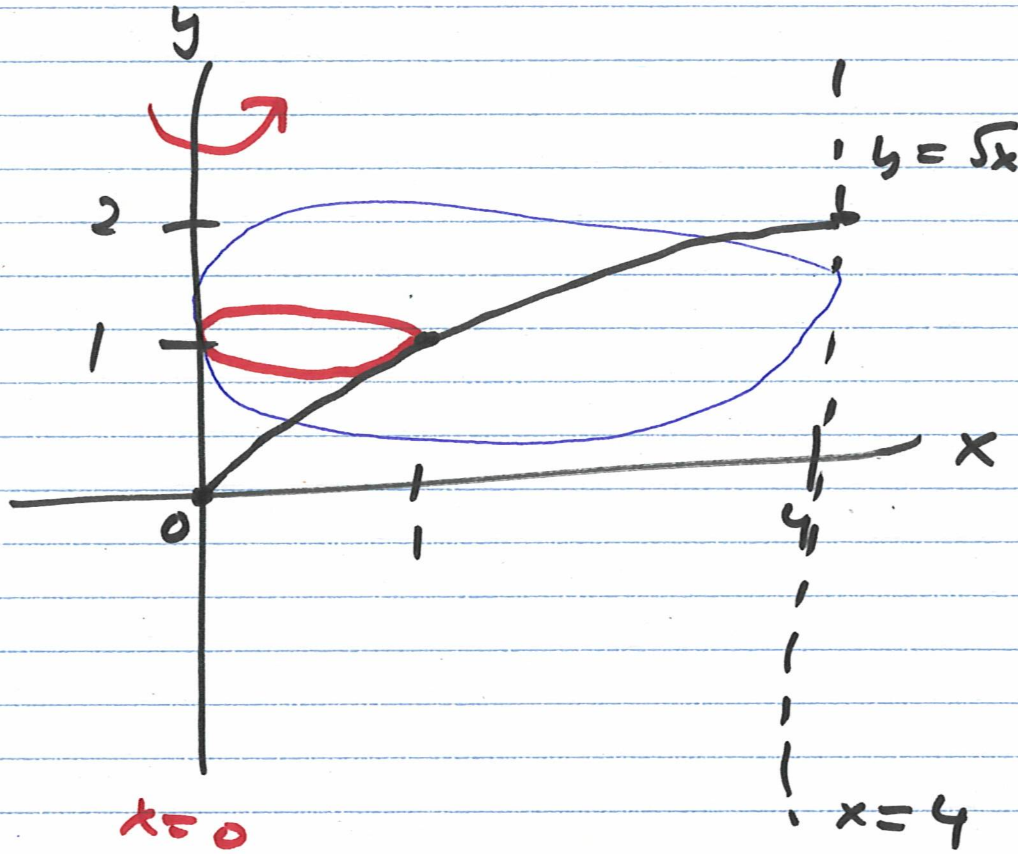
disk method $\quad V = \pi \displaystyle\int_c^d \left(g(y) \right)^2 dy$

Rotation Axis	Slice direction	Integration Var.
x-axis ($y=0$)	$\perp$ (vertical)	x
y-axis ($x=0$)	$\perp$ (horizontal)	y
$x = a$ (vertical)	$\perp$ (horizontal)	y
$y = b$ (horizontal)	$\perp$ (vertical)	x

Ex.1 Find volume of solid for region bounded by

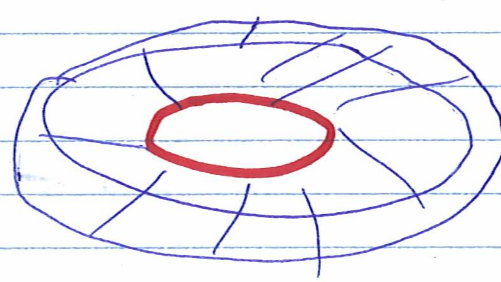
$y = \sqrt{x}$, $y = 0$, $x = 4$

rotated about the y-axis. $(x=0)$



y-axis: $x=0$ is vertical axis.

Washer is horizontal.



Washer:

Integration variable : $\boxed{y}$ = horizontal washer

$R(y) = 4 - 0 = 4$

$r(y) = y^2 - 0 = y^2$

$y = \sqrt{x} \iff x = y^2.$

Set : $\boxed{y^2 = 4} \iff y = \pm 2.$

but $-2 = y$ is extraneous.

$\boxed{y = 0, 2}$

<u>Washer</u>. $V = \pi \int_0^2 [R(y)]^2 - [r(y)]^2 \, dy$

$= \pi \int_0^2 4^2 - (y^2)^2 \, dy$

$= \pi \int_0^2 16 - y^4 \, dy$

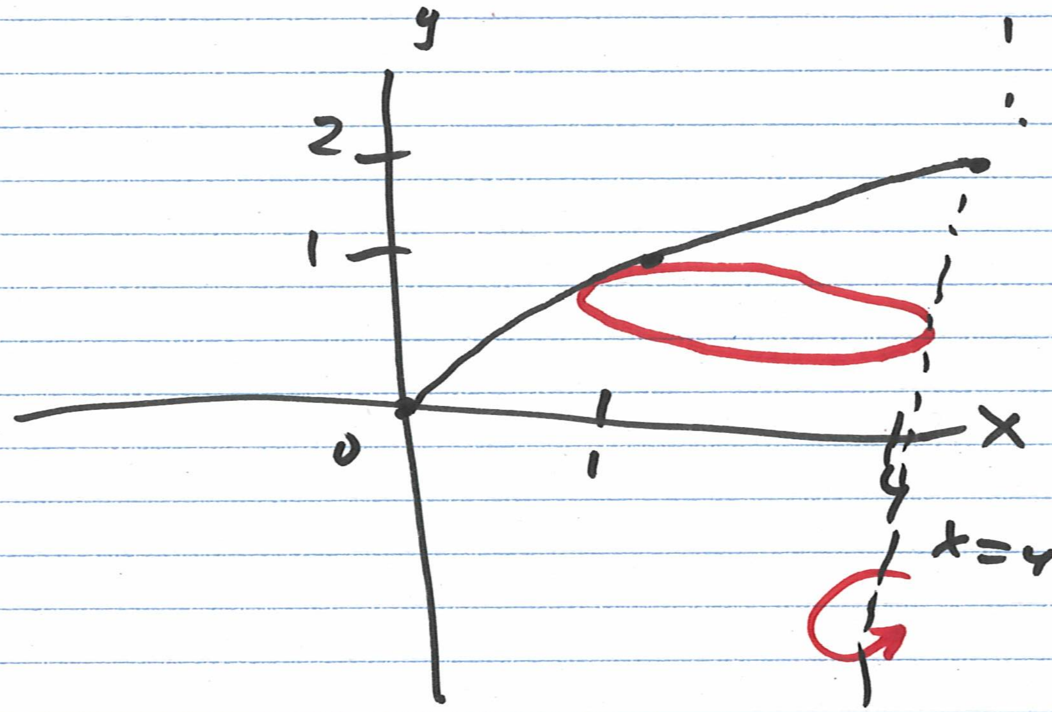
$= \pi \left[16y - \frac{y^5}{5} \right] \Big|_0^2$

$= \pi \left[32 - \frac{32}{5} \right] = \boxed{\frac{128\pi}{5}}$

Ex. 2 Find the volume of the solid of same region

$y = \sqrt{x}$, $y = 0$, $x = 4$, but

rotated about **vertical line** $x = 4$.



$x = 4$: vertical axis

horizontal disk:

disk.

$R(y) = 4 - y^2$

$y = \sqrt{x} \iff x = y^2$

$$V = \pi \int_0^2 (4 - y^2)^2 \, dy$$

$$= \pi \int_0^2 16 - 8y^2 + y^4 \, dy$$

$$= \pi \left[16y - \frac{8y^3}{3} + \frac{y^5}{5} \right] \Big|_0^2$$

$$= \pi \left[32 - \frac{64}{3} + \frac{32}{5} \right]$$

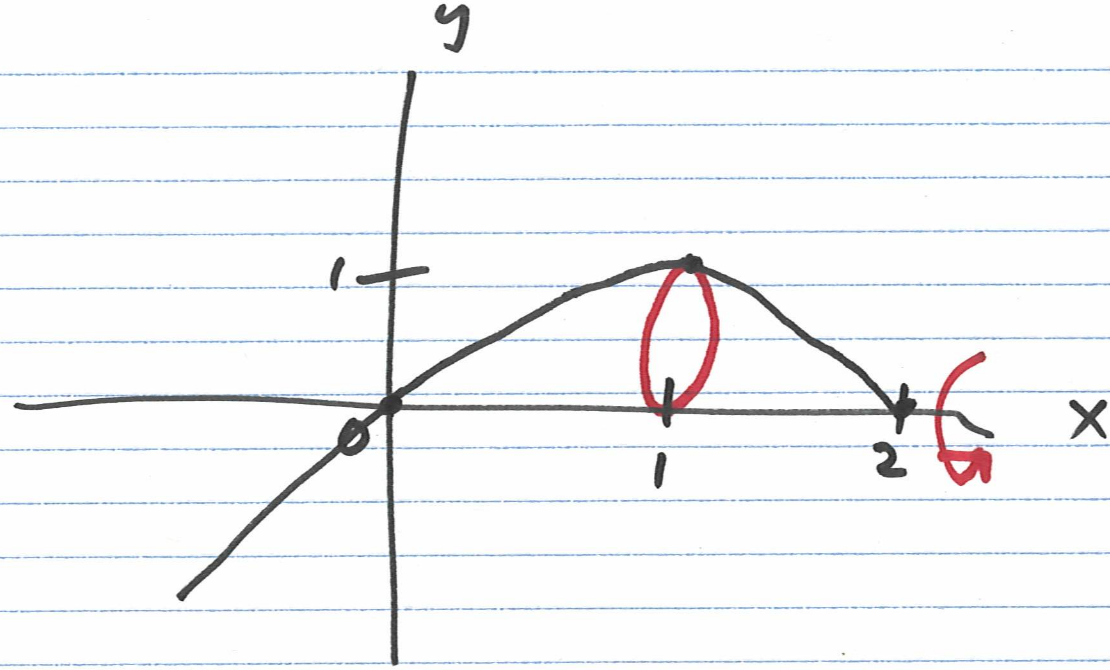
$$= \frac{32(15) - 64(5) + 32(3)}{15}$$

$$= \boxed{\frac{256\pi}{15}}$$

<u>Ex. 3</u> Find the volume of the solid obtained by rotating the region between the graphs of

$$y = x\sqrt{2-x} \quad \text{and} \quad y = 0.$$

around the x-axis.



x-axis (y=0): horizontal

↳ disk is vertical: (x)

$$R(x) = x\sqrt{2-x} - 0$$
$$= x\sqrt{2-x}$$

$$V = \pi \int_0^2 \left(x\sqrt{2-x}\right)^2 dx$$

$$= \pi \int_0^2 x^2(2-x)\, dx$$

$$= \pi \int_0^2 2x^2 - x^3\, dx$$

$$= \pi\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]\Bigg|_0^2$$

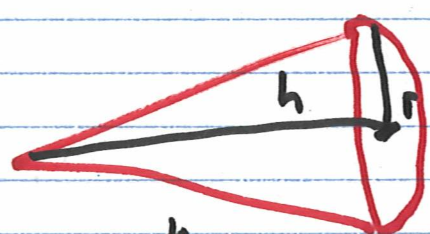
$$= \pi\left[\frac{16}{3} - \frac{16}{4}\right]$$

$$= \frac{\pi(64-48)}{12} = \frac{16}{12}\pi$$

$$= \boxed{\frac{4\pi}{3}}$$

Ex. Volume of a Cone: Disk Method:

Show: $V_{cone} = \frac{1}{3}\pi r^2 h$

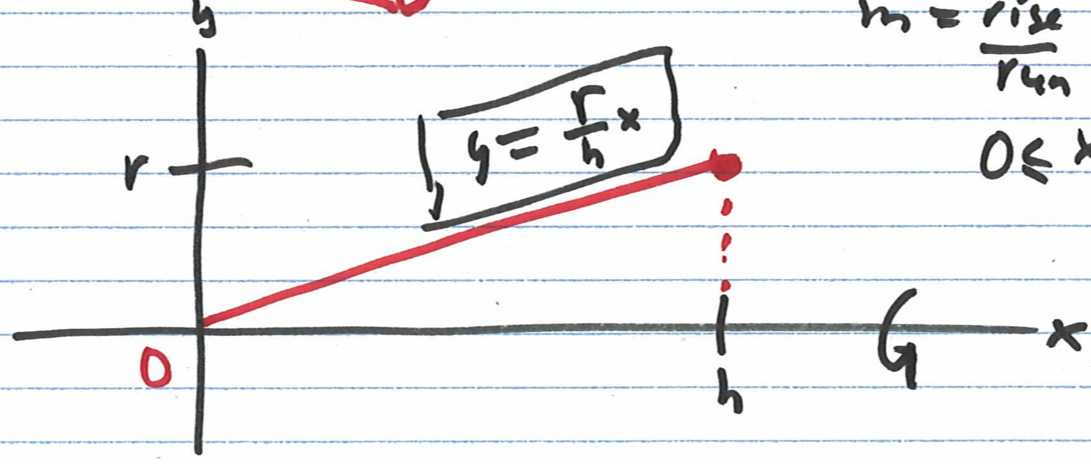


$y = mx + b$

$b = 0$

$m = \frac{rise}{run} = \frac{r}{h}$

$0 \le x \le h$



$$R(x) = \frac{r}{h}x.$$

<u>Disk</u>: $$V = \pi\int_0^h \left(\frac{r}{h}x\right)^2 dx$$

$$= \pi\int_0^h \frac{r^2}{h^2}x^2\,dx$$

$$= \pi\frac{r^2}{h^2}\int_0^h x^2\,dx$$

$$= \frac{\pi r^2}{h^2}\left.\frac{x^3}{3}\right|_0^h$$

$$= \frac{\pi r^2}{h^2}\frac{h^3}{3}$$

$$= \boxed{\frac{\pi r^2 h}{3}}.$$

$$\boxed{V_{cone} = \frac{1}{3}\pi r^2 h}$$