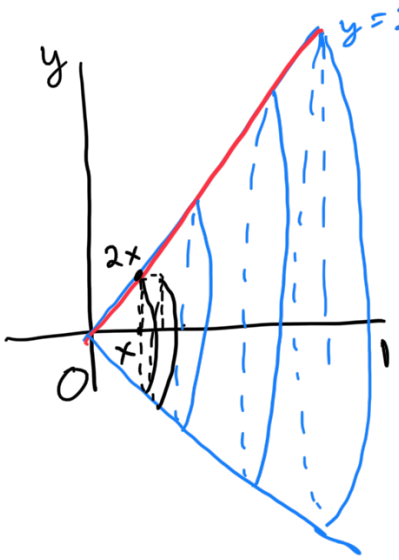
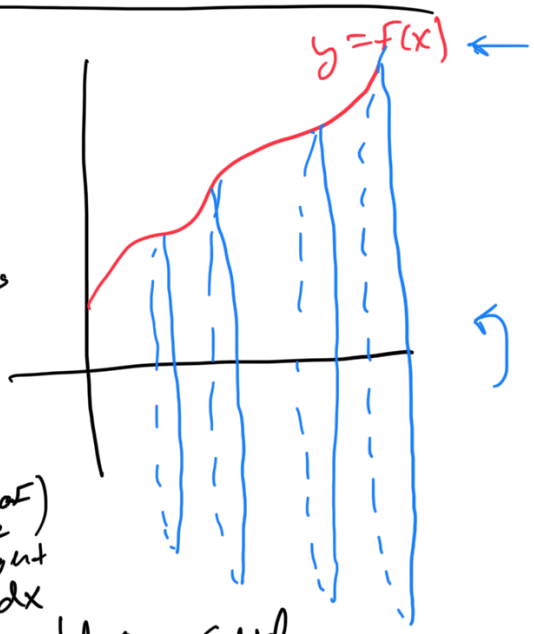
Lecture 3/15: Volumes of solids of revolution



volume = (area of base) · height

$= \pi (2x)^2 \cdot dx$



Ex: What is volume of the solid revolution formed by revolving $y = 2x$ on $[0,1]$ about x-axis?

$$\int_0^1 \pi (2x)^2 \, dx = 4\pi \int_0^1 x^2 dx$$

$$= 4\pi \frac{x^3}{3} \Big|_0^1 = \frac{4\pi}{3}$$

Disk method: Volume of solid of revolution formed by revolving $y = f(x)$ over $[a,b]$ about x-axis is:

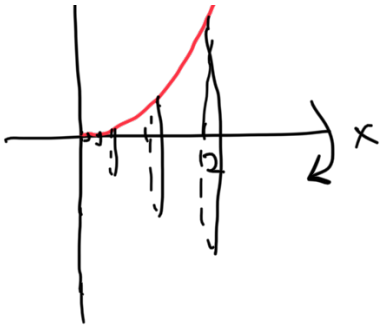
$$\int_a^b \pi (f(x))^2 \, dx.$$

Ex Revolve $y = x^2$ over $[0,2]$ about x-axis

b) $y = x^2$

From disk method:

$$\text{volume} = \int_0^2 \pi (x^2)^2 dx$$

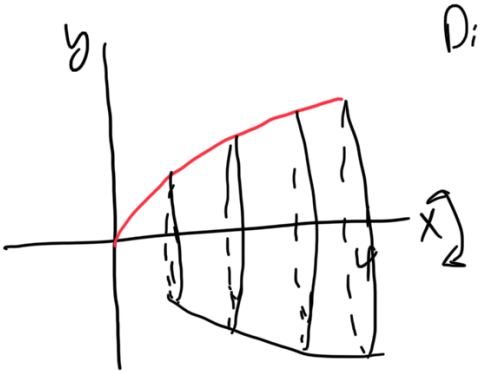


$$= \int_0^2 \pi x^4 dx$$

$$= \frac{\pi}{5} x^5 \Big|_0^2$$

$$= \frac{32\pi}{5}$$

Poll: Find volume of solid formed by revolving $y = \sqrt{x}$ over $[1, 4]$ about x-axis.



Disk method:

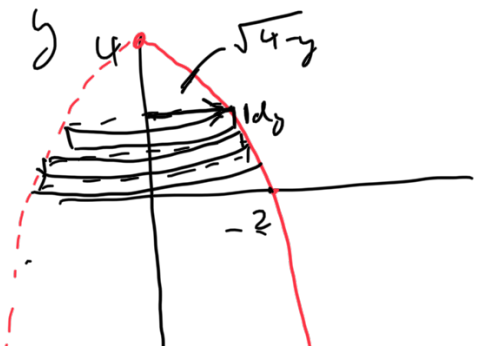
$$\int_1^4 \pi (\sqrt{x})^2 dx$$

$$= \int_1^4 \pi x \, dx$$

$$= \pi \frac{x^2}{2} \Big|_1^4$$

$$= \pi \frac{16}{2} - \pi \left(\frac{1}{2}\right) = \boxed{\frac{15\pi}{2}}$$

Ex: Find volume of solid formed by revolving $x = \sqrt{4-y}$ over $[0, 4]$ about y-axis.



Disk method:

$$\text{Volume} = \int_0^4 \pi (\sqrt{4-y})^2 dy$$

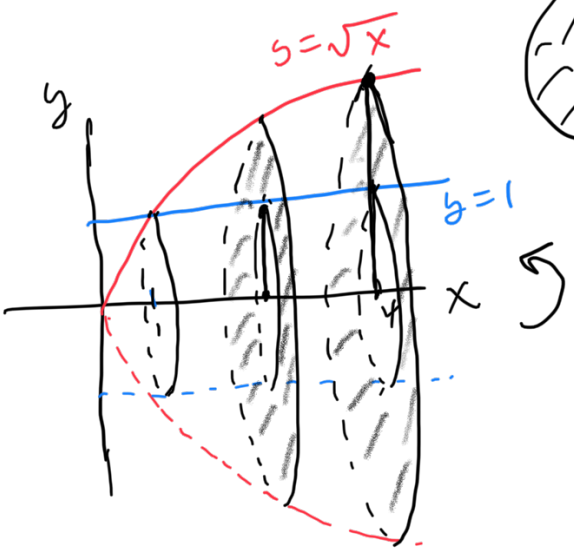
$$= \pi \int_0^4 (4-y) dy$$

$$= \left(4\pi y - \frac{\pi y^2}{2}\right) \Big|_0^4$$

$= 16\pi - 8\pi = 8\pi$

Solids formed by revolving region between two curves.

Ex



Washer
area $= \pi R^2 - \pi r^2$

Thickened washer
volume $= (\pi R^2 - \pi r^2)\,dx$

$R = \sqrt{x}$
$r = 1$

What is volume of solid formed by revolving region between $y = \sqrt{x}$ and $y = 1$ over $[1, 4]$ about the x-axis

$$\text{volume} = \int_1^4 \pi\left((\sqrt{x})^2 - 1^2\right) dx$$

$$= \pi \int_1^4 (x - 1)\,dx = \pi\left(\frac{x^2}{2} - x\right)\Big|_1^4$$

$$= \pi(8 - 4) - \pi\left(\frac{1}{2} - 1\right)$$

$$= 4\pi + \frac{\pi}{2}$$

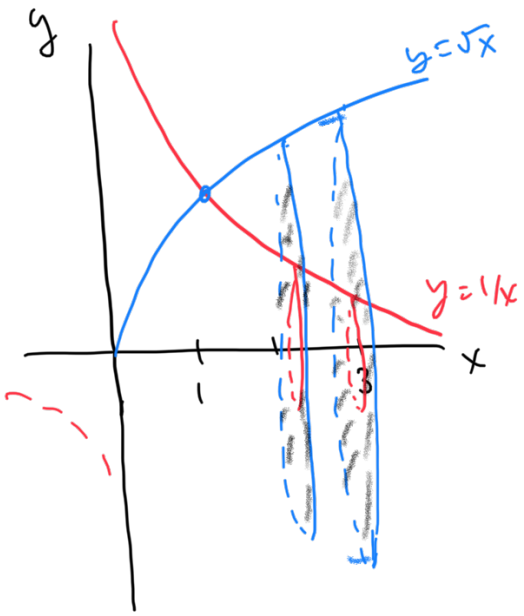
$$= \frac{8\pi + \pi}{2} = \frac{9\pi}{2}$$

Washer Method

If $F(x), g(x)$ functions with $F(x) \geq g(x)$ for x in $[a,b]$, then volume of solid formed by revolving region between $F(x), g(x)$ over $[a,b]$ about x-axis is

$$\int_a^b \pi \left((F(x))^2 - (g(x))^2 \right) dx.$$

Poll Find volume of solid formed by revolving region bounded by $\sqrt{x}$ and $1/x$ over interval $[1,3]$ about x-axis.



Intersection point:

Solve $\sqrt{x} = 1/x$

$x^{1/2} = 1/x$

$x^{3/2} = 1$

So $\sqrt{x} \geq 1/x$ for x in $[1,3]$

Volume

$$= \int_1^3 \pi \left((\sqrt{x})^2 - \left(\frac{1}{x}\right)^2 \right) dx$$

$$= \pi \int_1^3 \left(x - \frac{1}{x^2} \right) dx \qquad \frac{1}{x^2} = x^{-2}$$

$$= \pi \left(\frac{x^2}{2} + x^{-1} \right) \Big|_1^3$$

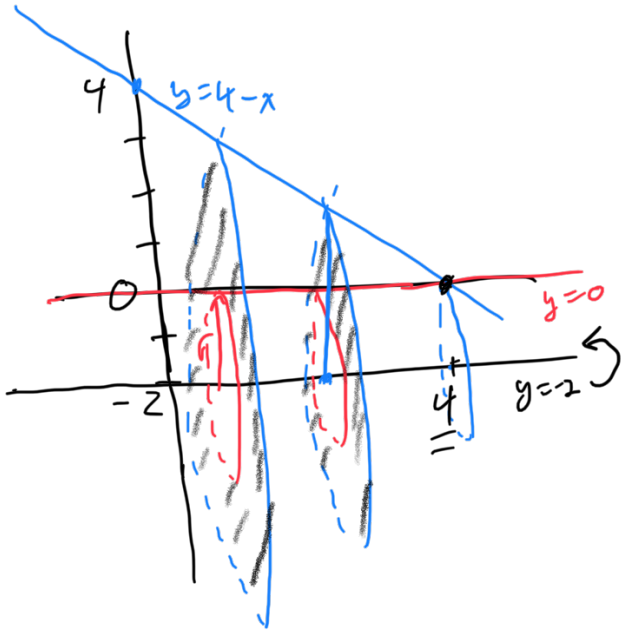
$$= \pi \left(a \quad \frac{1}{\quad} \right) - \pi \left(\frac{1}{\quad} + 1 \right)$$

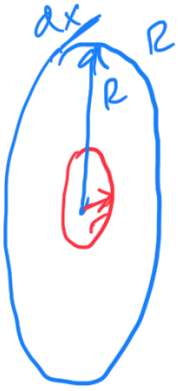
$$= \pi\left(\frac{9}{2} - \frac{1}{2} + \frac{1}{3} - 1\right)$$

$$= \pi\left(4 - 1 + \frac{1}{3}\right)$$

$$= \pi\left(3 + \frac{1}{3}\right) = \frac{10}{3}\pi$$

Ex: Find volume of solid formed by revolving region bounded by $4 - x$ and the x-axis about the line $y = -2$, over $[0, 4]$





$R = 4 - x - (-2)$

$= 6 - x$

$r = 2$

Volume of thickened washer

$$= \pi\left((6-x)^2 - 2^2\right) dx$$

$$\text{Volume} = \int_0^4 \pi\left((6-x)^2 - 2^2\right) dx$$

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