Example 3: The velocity of a car over the time period $0 \le t \le 3$ is given by the function

$$v(t) = 50 + e^{-t/4} \text{ miles per hour}$$

where t is time in hours. What was the distance the car traveled in the first 30 minutes?

Note t is in hours. So 30 mins $\Rightarrow$ 0.5 hrs

We need to compute

$$\int_0^{0.5} 50 + e^{-t/4}\,dt \qquad \begin{matrix} u = 50t & dv = e^{-t/4}\,dt \\ du = 50\,dt & v = -4e^{-t/4} \end{matrix} \qquad uv - \int v\,du$$

$$= 50t(-4e^{-t/4})\Big]_0^{0.5} - \int_0^{0.5} -4e^{-t/4}(50)\,dt$$

$$= -200te^{-t/4}\Big]_0^{0.5} + 200\int_0^{0.5} e^{-t/4}\,dt$$

$$= -200te^{-t/4}\Big]_0^{0.5} + 200(-4)e^{-t/4}\Big]_0^{0.5}$$

$$= -200(0.5)e^{-0.5/4} - (-200(0)e^{-0/4}) - 800e^{-0.5/4} - (-800e^{-0/4})$$

$$= -100e^{-1/8} - 0 - 800e^{-1/8} + 800$$

$$= -900e^{-1/8} + 800$$

$$\approx 5.75$$

Example 4: A model for the ability of a child to memorize information, measured on a scale from 1 to 100, is given by

$$M(t) = 1 + 3.4t + t\ln(t)$$

when $2 \le t \le 8$ where t is the child's age in years. Find the child's average memorization ability between ages 3 and 6.

$$M_{AVE}(t) = \frac{1}{6-3}\int_3^6 (1 + 3.4t + t\ln(t))\,dt$$

$$= \frac{1}{3}\int_3^6 dt + \frac{3.4}{3}\int_3^6 t\ln(t)\,dt$$

$u = \ln(t) \quad dv = t\,dt$

$du = \frac{1}{t}dt \quad v = \frac{t^2}{2}$

$$= \frac{1}{3}\int_3^6 dt + \frac{3.4}{3}\left(\frac{t^2}{2}\ln(t)\right]_3^6 - \int_3^6 \frac{t^2}{2}\cdot\frac{1}{t}\,dt\Big)$$

$$= \frac{1}{3}\int_3^6 dt + \frac{3.4}{3}\left(\frac{t^2}{2}\ln(t)\right]_3^6 - \int_3^6 \frac{1}{2}t\,dt\Big)$$

$$= \frac{1}{3}t\Big]_3^6 + \frac{3.4}{6}t^2\ln(t)\Big]_3^6 - \frac{3.4}{6}\cdot\frac{t^2}{2}\Big]_3^6$$

$$= \frac{1}{3}(6) - \frac{1}{3}(3) + \frac{3.4}{6}(6)^2\ln(6) - \frac{3.4}{6}(3)^2\ln(3) - \frac{3.4}{12}(6)^2 - \left(\frac{-3.4}{12}(3)^2\right)$$

≈ 26.299