MA 16020: Lesson 18
Volume By Revolution
Shell Method
Pt 2: Rotation around any non-Axis

By General Ozochiawaeze

RECAP: When should I use Shell Method? How do I use Shell Method?

If you find **solving for x or y**, for either Disk or Washer Method, **is hard**

$\Rightarrow$ **Shell Method**

For rotation around x-axis:

$$V = 2\pi \int_c^d y \cdot \left(Right - Left\right) dy$$

For rotation around y-axis:

$$V = 2\pi \int_a^b x \cdot (Top - Bottom) dx$$

		Axis of Rotation	
		x-axis	y-axis
Method	Disk/Washer	dx	dy
	Shells	dy	dx

What happens if we are revolving around non-Axes (like $x = a$ or $y = b$) ?

Answer: everything stays the same except for the **RADIUS**.

How did I find the radius, $r(x)$, last class?

- **To find $r(x)$, we need to find the distance of the shell from the axis of rotation**

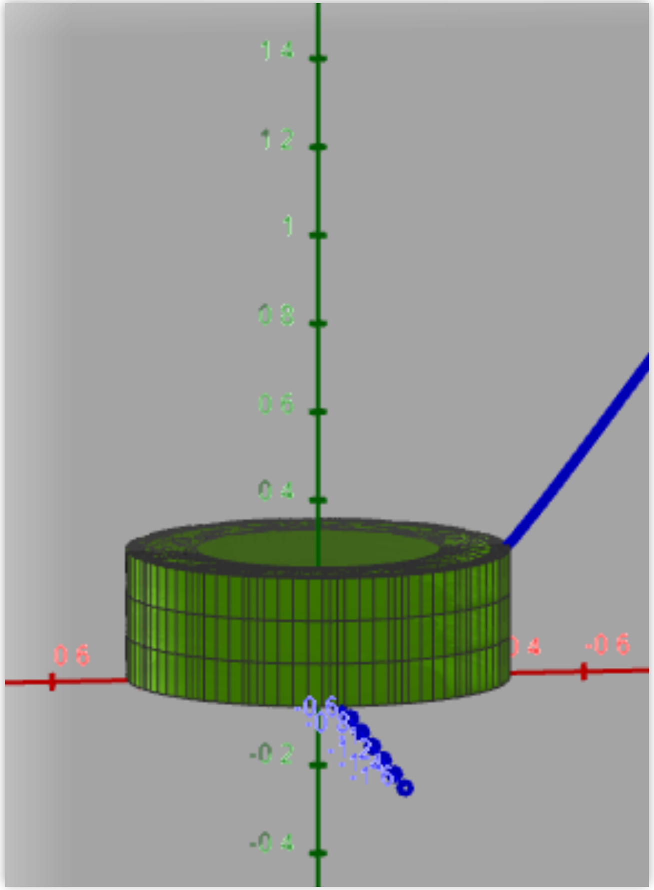
- The shell is x units away from the y-axis. So,

  $r(x) = x$

- So, the formula:

$$V = 2\pi \int_a^b r(x) \cdot (Top - Bottom)dx$$

$$V = 2\pi \int_a^b x \cdot (Top - Bottom)dx$$



https://www.geogebra.org/m/jqfyndpu

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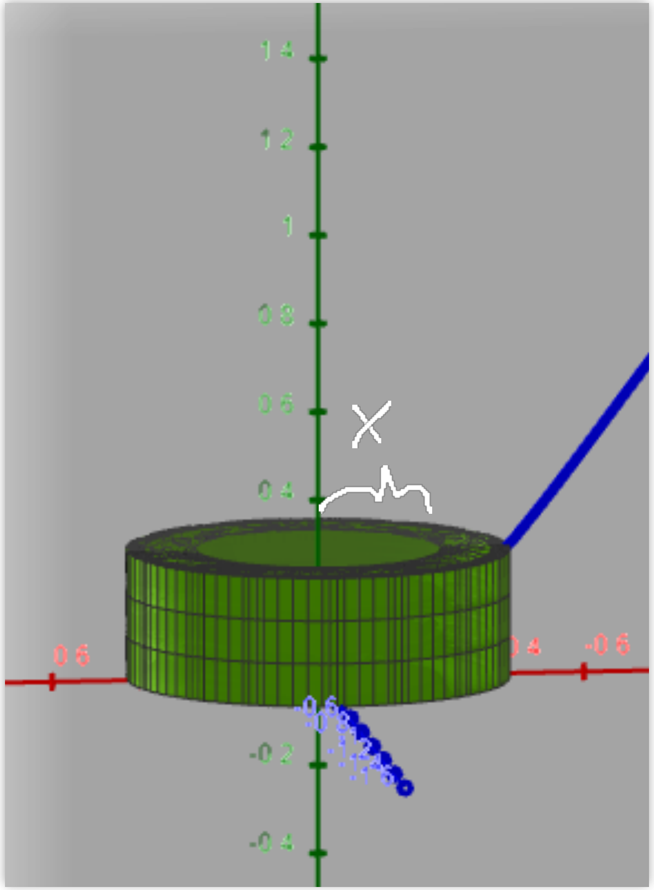

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What is $r(x)$ when we are rotating around something other than y-axis?

- **Again, to find $r(x)$, we need to find the distance of the shell from the axis of rotation**

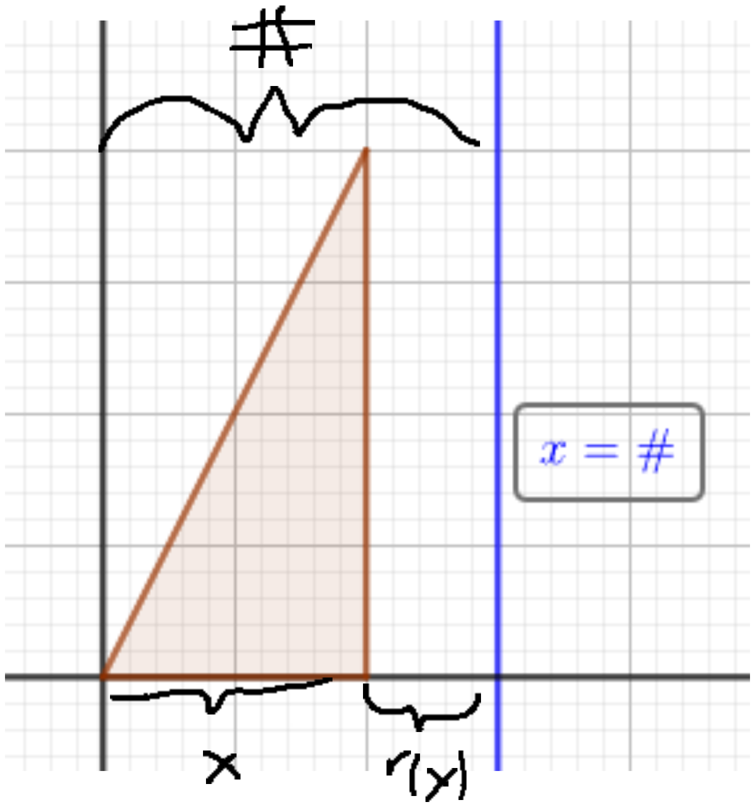
- From the picture we see, # is the distance from the y-axis and the line of rotation. So,

$$x + r(x) = \# \qquad \Rightarrow \qquad r(x) = \# - x$$

- If the axis of rotation is on the right of your region,

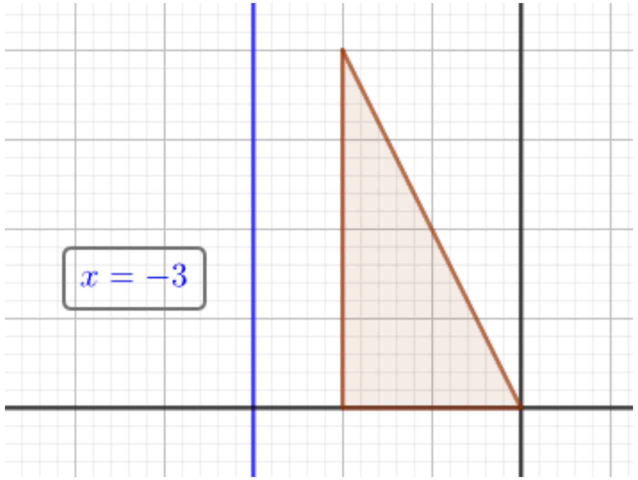
$$V = 2\pi \int_a^b r(x) \cdot (Top - Bottom)dx$$

$$V = 2\pi \int_a^b (\# - x) \cdot (Top - Bottom)dx$$



What if the axis of rotation is on the left of the region? Is it the same formula?

Take the example below:



- If $r(x) = \# - x,$ then $r(x) = -3 - x.$
  - Choose a value in the region (i.e. triangle).
  - Is r(x) still positive? **No.**

What if $r(x) = x - \#$? **Yes.**

- **Almost…. If the axis of rotation $x = \#$ is on the left, then**

$\boldsymbol{r(x) = x - \#}$

  - Why? The radius needs to always positive

- If the axis of rotation is on the left of your region,

$$V = 2\pi \int_a^b r(x) \cdot (Top - Bottom)dx$$

$$V = 2\pi \int_a^b (x - \#) \cdot (Top - Bottom)dx$$

Overall, the Shell Method Formulas around any non-axis are…

- Rotating around $x = \#$
  - If the axis of rotation is on the right of your region,

$$V = 2\pi \int_a^b (\# - x) \times (Top - Bottom)\, dx$$

  - If the axis of rotation is on the left of your region,

$$V = 2\pi \int_a^b (x - \#) \times (Top - Bottom)\, dx$$

- Rotating around $y = \#$
  - If the axis of rotation is above of your region,

$$V = 2\pi \int_a^b (\# - y) \times (Right - Left)\, dy$$

  - If the axis of rotation is below of your region,

$$V = 2\pi \int_a^b (y - \#) \times (Right - Left)\, dy$$

Example 1: Using the **Shell Method,** set up the integral that represents the volume of solid obtained by revolving the region defined by

a triangle with vertices at $(0,0)$, $(2,0)$, and $(2,3)$

about the line indicated

a) about $x = 3$

https://www.geogebra.org/m/knnfvjub#material/abjaakfk

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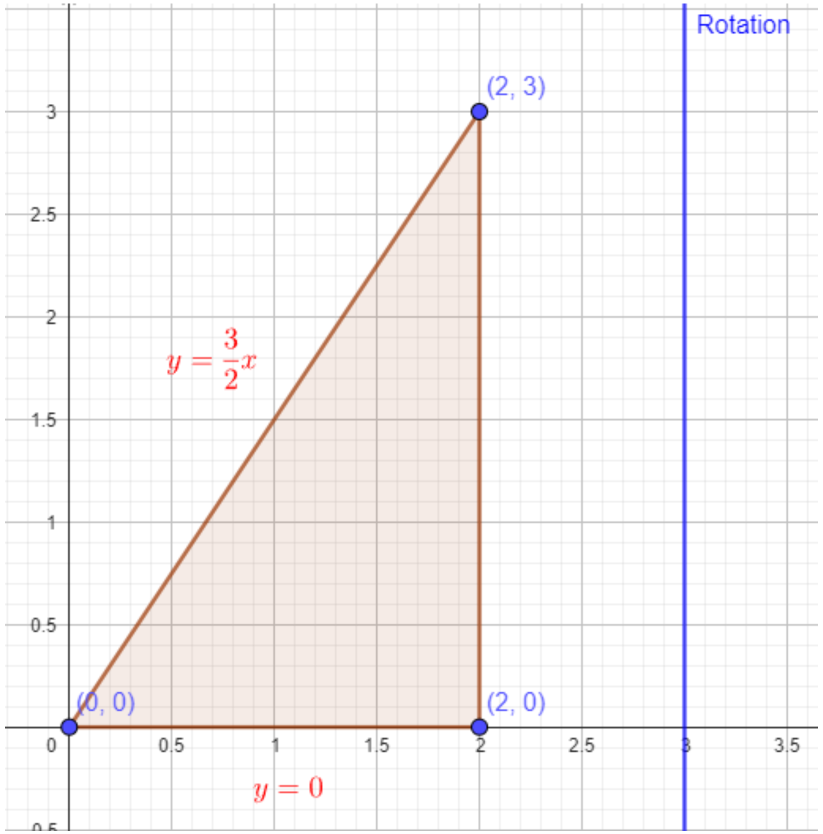
Draw the region.

a) about $x = 3$



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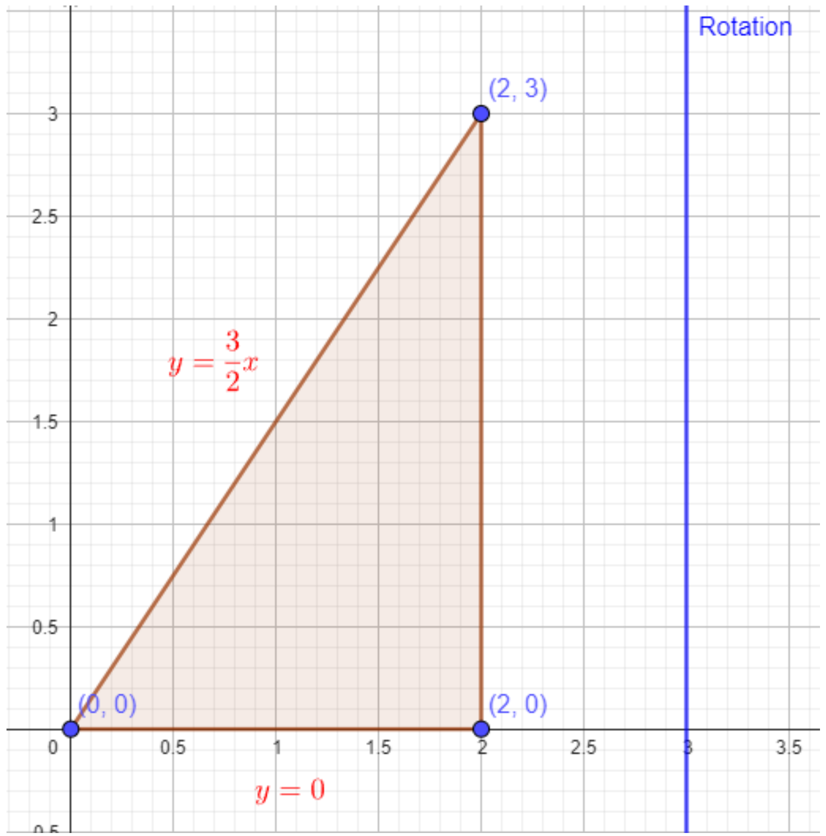

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b) about $x = -1$

https://www.geogebra.org/m/knnfvjub#material/kcw32ctc

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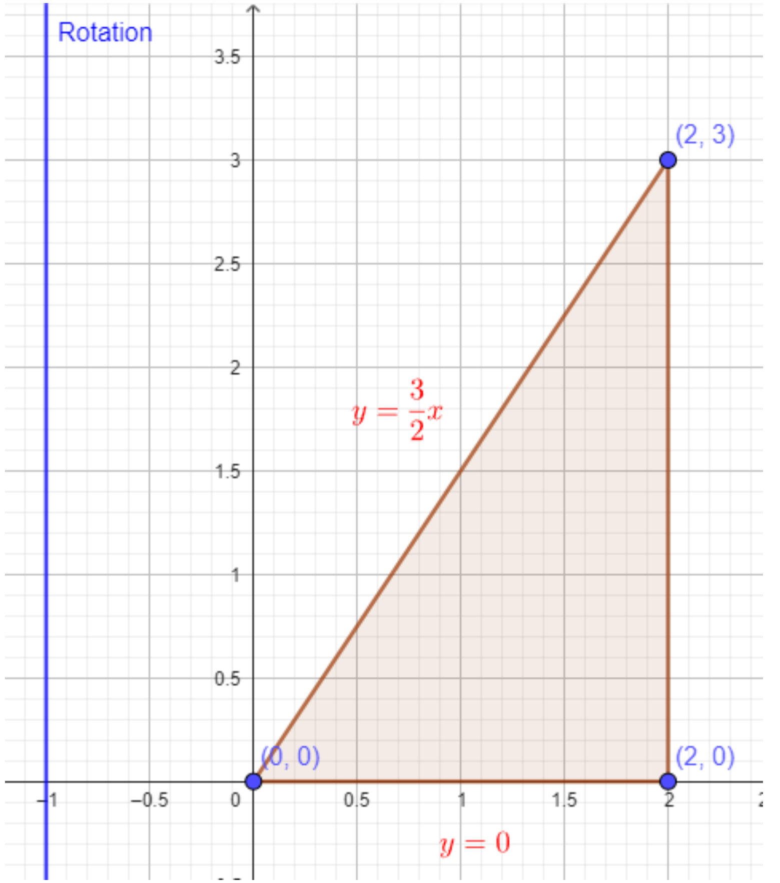

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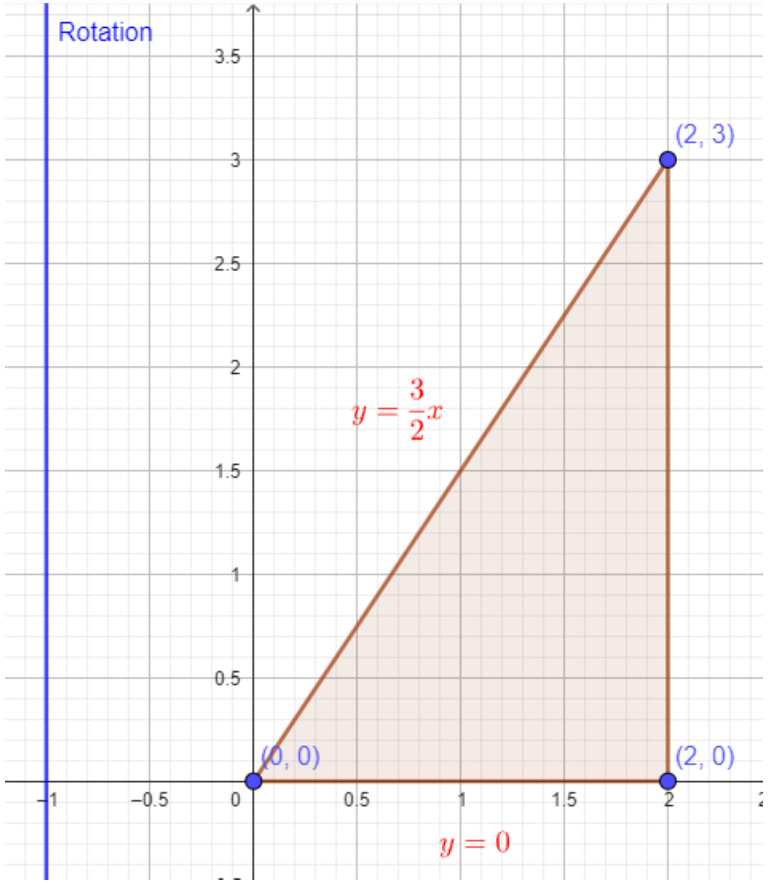

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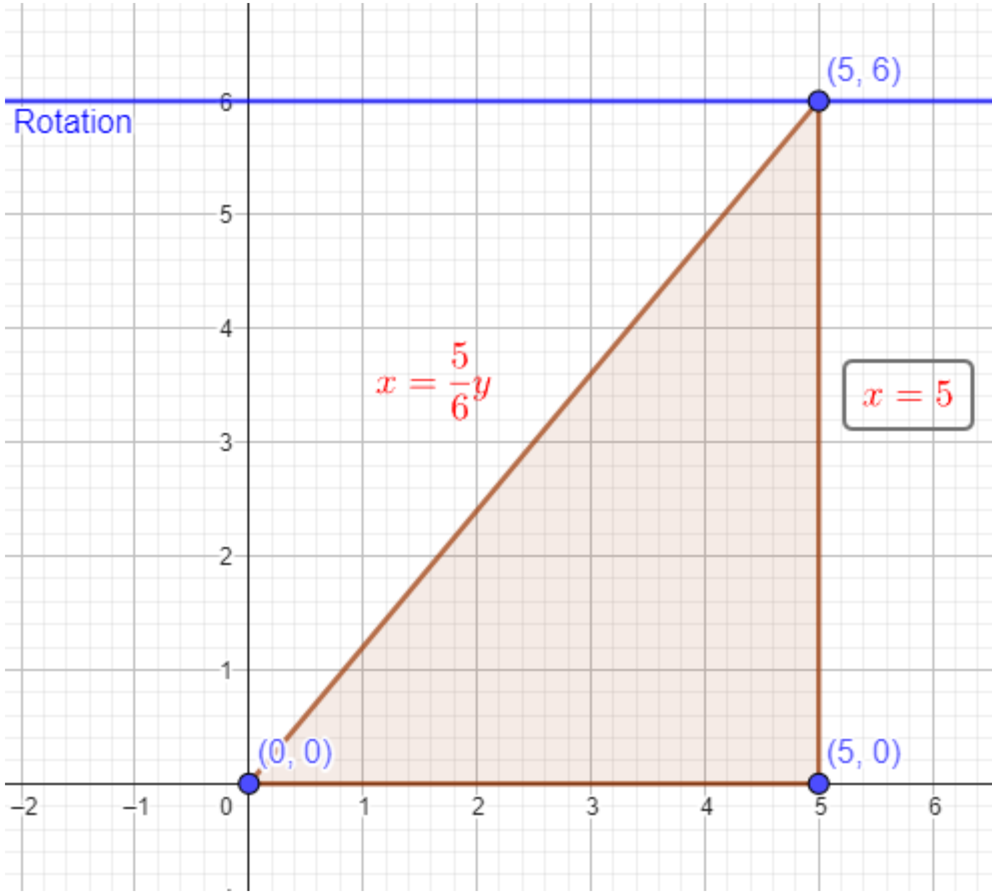
Example 2: Using the **Shell Method,** set up the integral that represents the volume of solid obtained by revolving the region defined by

a triangle with vertices at $(0,0)$, $(5,0)$, and $(5,6)$

about the line indicated

Draw the region.

a) About $y = 6$



https://www.geogebra.org/m/knnfvjub#material/e7hwecfx

Example 2: Consider the region bounded by:

$$y = \sqrt{x}\,, \qquad y = 0\,, \quad \text{and} \quad x = 4$$

Set up the integral that represents the volume of solid obtained by the rotating the region using the **Shell Method**

a) about $x = 5$

https://www.geogebra.org/m/knnfvjub#material/ajknmy46

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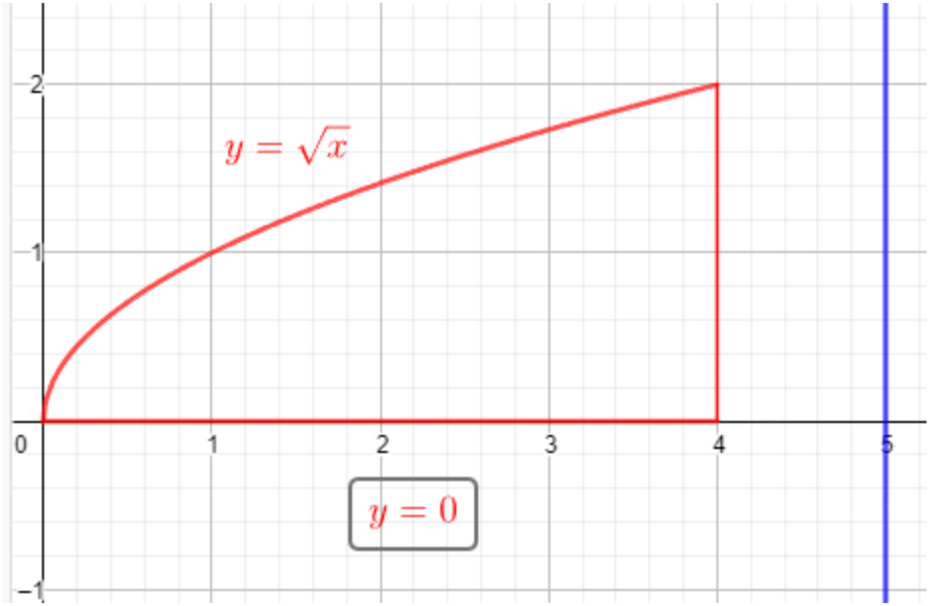

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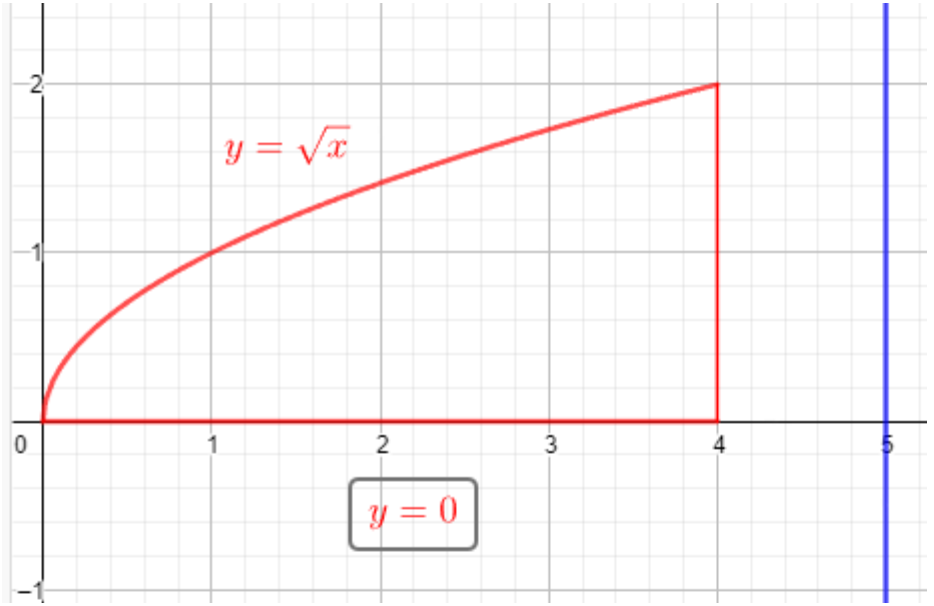

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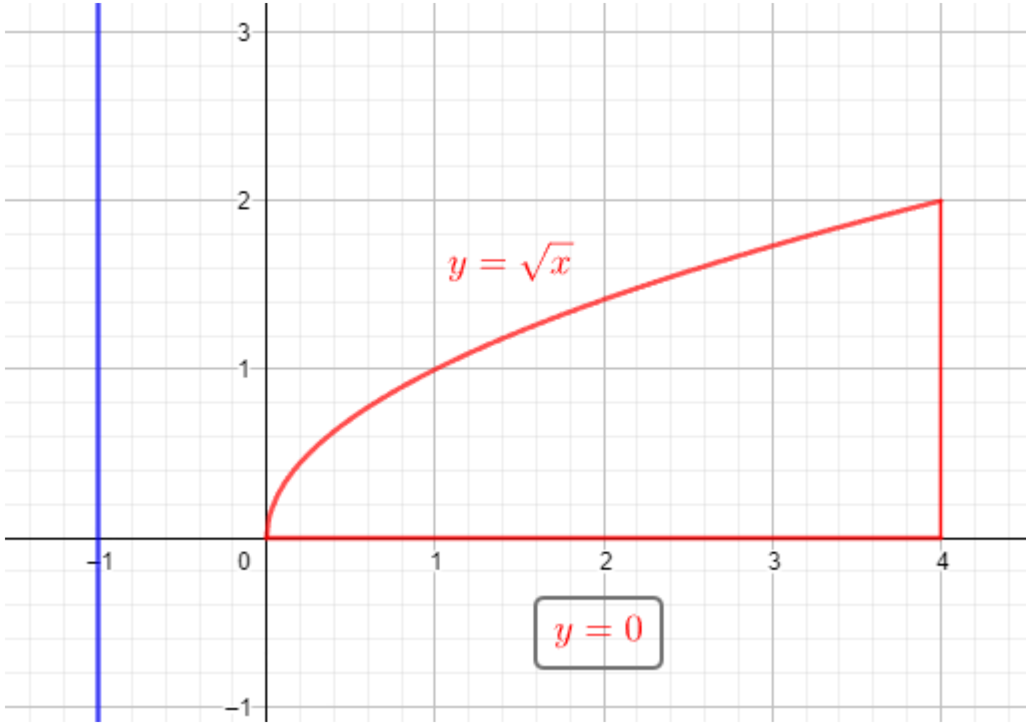

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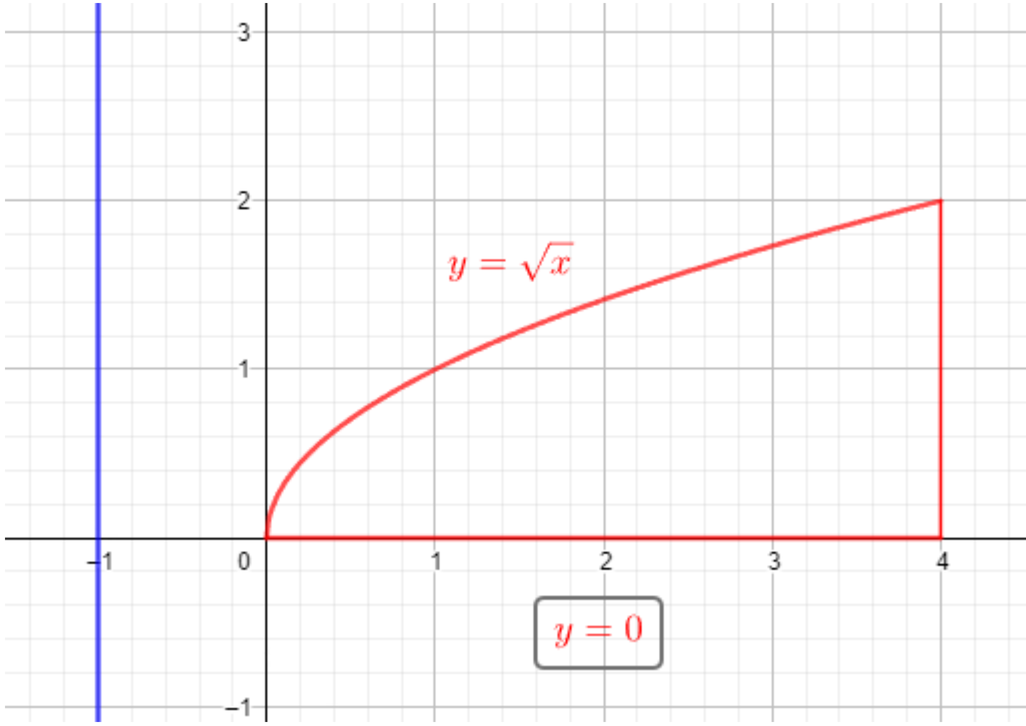

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Example 3: Consider the region bounded by:

$$x = y^2 + 1, \qquad \text{and} \qquad x = 2$$

Set up the integral that represents the volume of solid obtained by the rotating the region using the **Shell Method**

a) about $y = 3$

https://www.geogebra.org/m/knnfvjub#material/agxjnwfd

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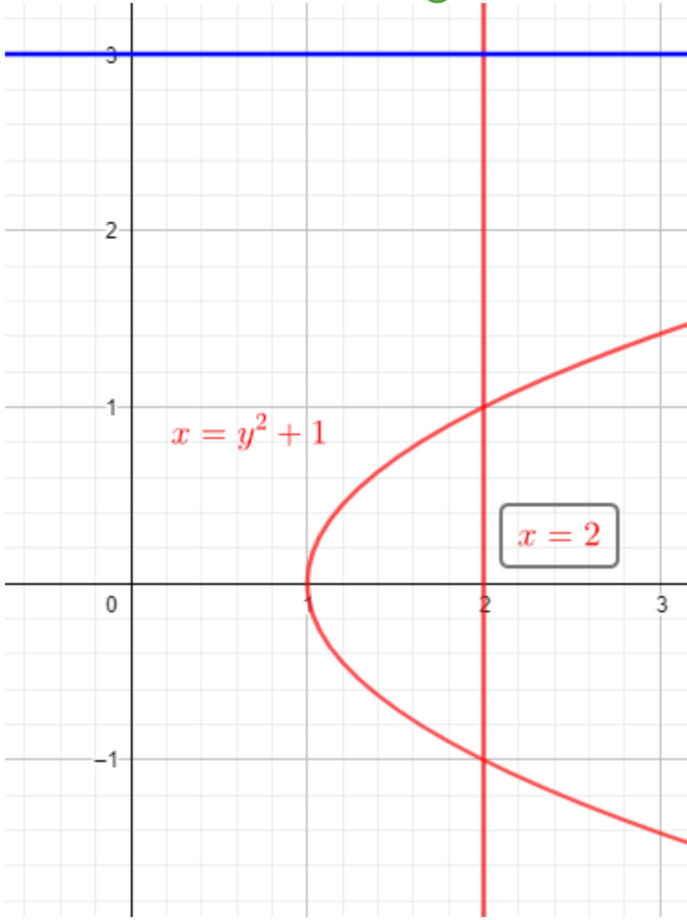

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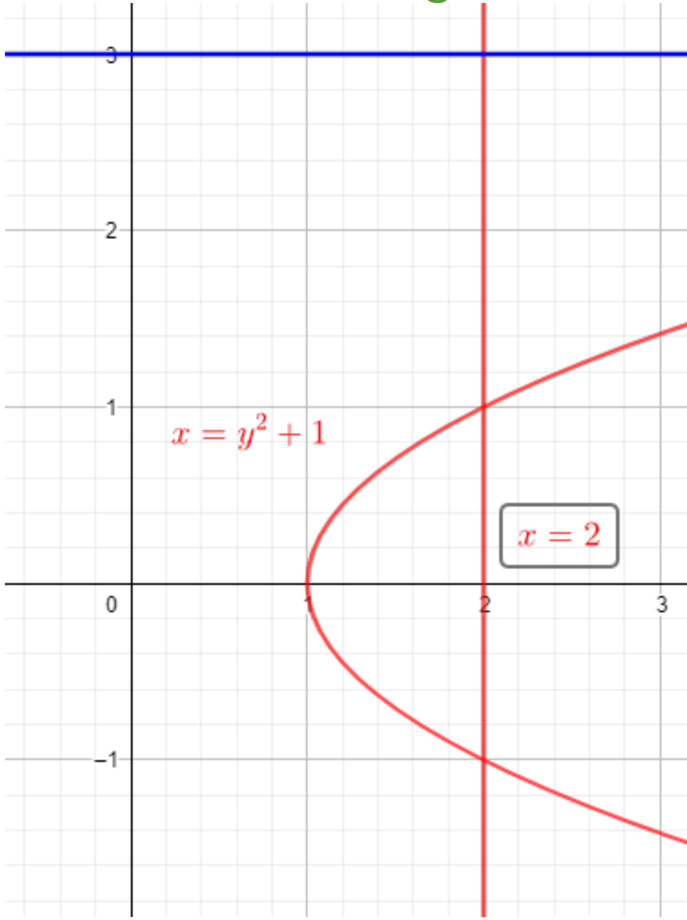

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b) about $y = -2$

https://www.geogebra.org/m/knnfvjub#material/hjhswuca

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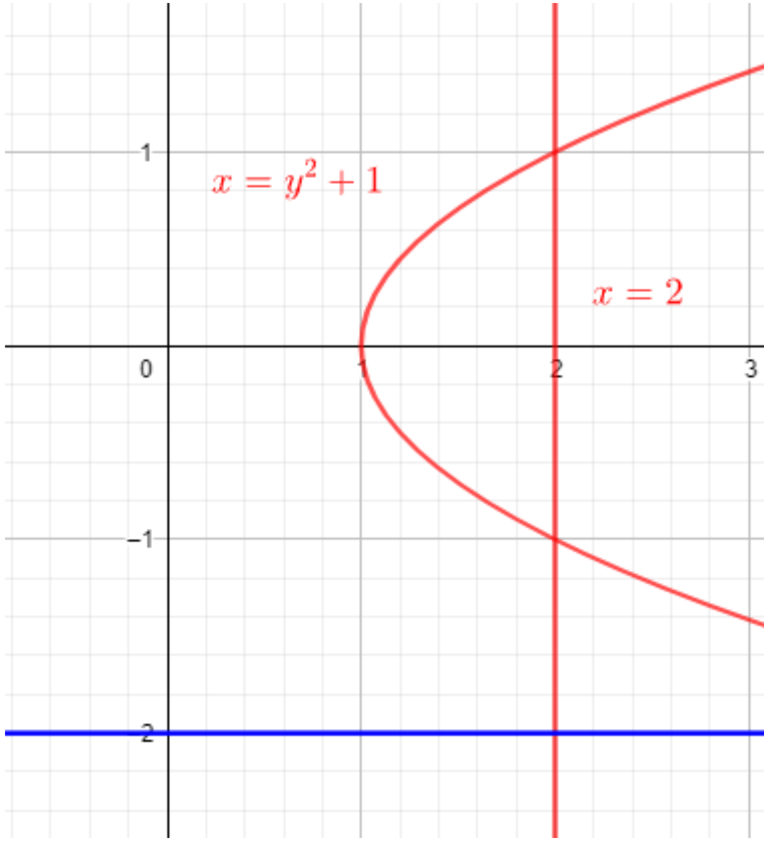

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When do we apply Disk Method or Washer Method or Shell Method?

- When the region **“hugs”** the axis of rotation
  ⇒ **Disk Method**

- When there is a **“gap”** between the region and axis of rotation
  ⇒ **Washer Method**

- But if you find **solving for x or y**, in either method, **is hard**
  ⇒ **Shell Method**

		Axis of Rotation	
		x-axis or	y-axis or
Method	Disk/Washer	dx	dy
	Shells	dy	dx

Formulas from Lessons 14 and 15 and 17
Rotation around x-axis or y-axis

For rotation around x-axis:

- Disk Method:

$$V = \pi \int_a^b [f(x)]^2 \, dx$$

- Washer Method:

$$V = \pi \int_a^b \left(R^2 - r^2\right) dx$$

- Shell Method:

$$V = 2\pi \int_c^d y \cdot (Right - Left) \, dy$$

For rotation around y-axis:

- Disk Method:

$$V = \pi \int_c^d [g(y)]^2 \, dy$$

- Washer Method:

$$V = \pi \int_c^d \left(R^2 - r^2\right) dy$$

- Shell Method:

$$V = 2\pi \int_a^b x \cdot (Top - Bottom) \, dx$$

Formulas from Lesson 15 and 18
Rotation around any non-Axis Formulas

For rotation around the line $x = \#$:

- Disk Method:

$$V = \pi \int_a^b [f(x) - \#]^2 \, dx$$

- Washer Method:

$$V = \pi \int_a^b \left[(R - \#)^2 - (r - \#)^2 \right] dx$$

- Shell Method:
  - If the axis of rotation is on the left of your region,

$$V = 2\pi \int_a^b (x - \#) \times (Top - Bottom) \, dx$$

  - If the axis of rotation is on the right of your region,

$$V = 2\pi \int_a^b (\# - x) \times (Top - Bottom) \, dx$$

Note: That these formulas work for the case of x-axis ($y = 0$) and y-axis ($x = 0$).

Formulas from Lesson 15 and 18
Rotation around any non-Axis Formulas

For rotation around the line $y = \#$:

- Disk Method:

$$V = \pi \int_c^d [g(y) - \#]^2 \, dy$$

- Washer Method:

$$V = \pi \int_c^d \left[(R - \#)^2 - (r - \#)^2 \right] dy$$

- Shell Method:
  - If the axis of rotation is below your region,

$$V = 2\pi \int_a^b (y - \#) \times (Right - Left) \, dy$$

  - If the axis of rotation is above your region,

$$V = 2\pi \int_a^b (\# - y) \times (Right - Left) \, dy$$

Note: That these formulas work for the case of x-axis ($y = 0$) and y-axis ($x = 0$).