# A linear sampling method for the inverse cavity scattering problem of biharmonic waves

## My Advanced Topics Presentation

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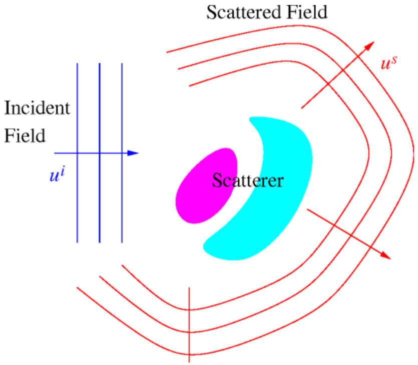
# Overview

- Part 1: *Introduction*
- Part 2: *Direct Scattering Problem for the Biharmonic Wave Equation*
- Part 3: *Direct Imaging Methods*
- Part 4: *Reconstruction of the Cavity* $D$ *via the Linear Sampling Method*
- Part 5: *Ongoing Future Work*

# Part 1: Introduction

# Inverse Obstacle Wave Scattering

- Send a wave and observe the reflected wave by an unknown obstacle
- **Question:** What information about the obstacle can one extract from the observed wave?
- **Type of waves:** *flexural waves in elastic plates* (biharmonic wave equation)



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# Applications of Biharmonic Wave Scattering

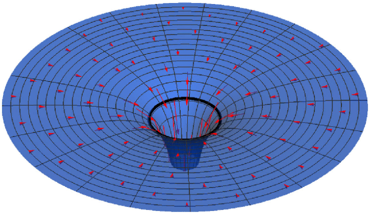

Figure: View of an Acoustic Black Hole: Technique for Passive Vibration Control
Wang, Q. & Ge, X. (2020)

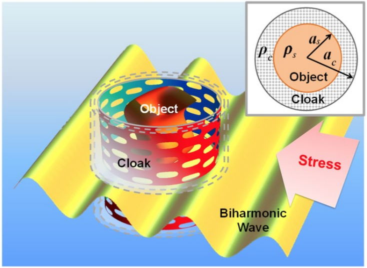


Figure: A cylindrical shell acting as a platonic elastic cloak of an object in a thin elastic plate
Farhat, M., Chen, PY., Bağcı, H. *et al*. (2014)



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# Applications of Biharmonic Wave Scattering

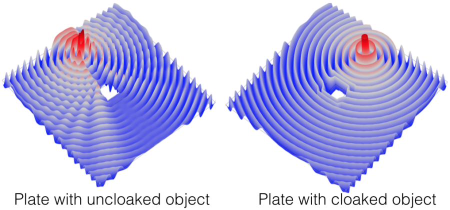


Figure: Elastic Cloaking
Colquitt, D. (2015)

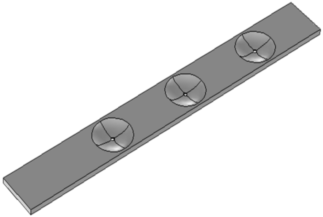

Figure: A schematic of a plate with three equally spaced neutralisers for vibration damping



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# Part 2: Direct Scattering Problem for the Biharmonic Wave Equation

### Problem (The Direct Scattering Problem)

*We consider the time-harmonic biharmonic scattering problem*

1. $D \subset B_\rho := \{x \in \mathbb{R}^2 \, : \, |x| \leq \rho\} \subset \mathbb{R}^2$ *is a clamped cavity with* $\partial D$*-Smooth*
2. *The cavity receives illumination from the incident plane wave* $u^i = \exp(ikx \cdot d)$
3. $\Gamma_\rho = \{x \in \mathbb{R}^2 \, : \, |x| = \rho\}$

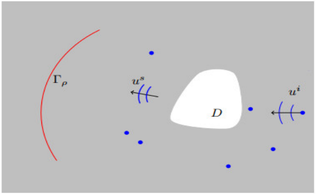


Figure: Clamped Cavity in a Thin Plate

# Direct Scattering of Biharmonic Waves

The total field $u = u^i + u^s \in H^2_{loc}(\mathbb{R}^2)$ satisfies, with $r = |x|$,

$$\begin{cases} \Delta^2 u - k^4 u = 0 \quad \text{in } \mathbb{R}^2 \setminus \overline{D} \\ u = 0, \quad \partial_n u = 0 \quad \text{on } \partial D \\ \lim\limits_{r\to\infty} r^{1/2} \left(\partial_r u^s - iku^s\right) = 0, \quad \lim\limits_{r\to\infty} r^{1/2} \left(\partial_r \Delta u^s - ik\Delta u^s\right) = 0 \end{cases} \tag{1}$$

**Remark**

*Let* $u^i = \exp(ikx \cdot d)$ *then the radiating scattered field* $u^s(x, d; k)$ *depends on the incident direction* $d$ *and wave number* $k$.

The scattered field, also known as the radiating solution, has the following asymptotic expansion

$$u^s(x,d;k) = \frac{e^{ikr}}{r^{1/2}} u^\infty(\hat{x}) + O\left(\frac{1}{r^{3/2}}\right) \quad \text{as } r = |x| \to \infty$$

where $\hat{x}, d \in \mathbb{S}^1 = \{x \in \mathbb{R}^2 \,:\, |x| = 1\}$.

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where $\hat{x}, d \in \mathbb{S}^1 = \{x \in \mathbb{R}^2 : |x| = 1\}$. Now define the far-field operator as $\mathcal{F} : L^2(\mathbb{S}^1) \to L^2(\mathbb{S}^1)$

$$(\mathcal{F}g)(\hat{x}) = A_g \coloneqq \int_{\mathbb{S}^1} u^\infty(\hat{x}, d; k) g(d)\, ds(d).$$



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$$(\mathcal{F}g)(\hat{x}) = A_g \coloneqq \int_{\mathbb{S}^1} u^\infty(\hat{x}, d; k) g(d)\, ds(d).$$

The **inverse problem** reads: Given $\mathcal{F}$ for a range of wave numbers obtain qualitative information about the cavity $D$ in a thin elastic plate.



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# Biharmonic Wave Decomposition

Consider the two auxiliary functions

$$u_H^s = -\frac{1}{2k^2}(\Delta u^s - k^2 u^s), \quad u_M^s = \frac{1}{2k^2}(\Delta u^s + k^2 u^s)$$

$u_H^s$ is the 'propagative part' of $u^s$ and $u_M^s$ is the 'evanescent part' of $u^s$ such that

$$u^s = u_H^s + u_M^s, \quad \Delta u^s = k^2(u_M^s - u_H^s)$$



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$$u^s = u_H^s + u_M^s, \quad \Delta u^s = k^2(u_M^s - u_H^s)$$

$u_H^s$ and $u_M^s$ satisfy the Helmholtz equation and modified Helmholtz equation respectively

$$\Delta u_H^s + k^2 u_H^s = 0, \quad \Delta u_M^s - k^2 u_M^s = 0 \quad \text{in } \mathbb{R}^2 \setminus \overline{D}$$



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# Biharmonic Wave Decomposition

We can reformulate the scattering problem (1) as

$$\begin{cases} \Delta u_H^s + k^2 u_H^s = 0, \quad \Delta u_M^s - k^2 u_M^s = 0 \quad \text{in } \mathbb{R}^2 \setminus \overline{D} \\ u_H^s + u_M^s = -u^i, \quad \partial_n u_H^s + \partial_n u_M^s = -\partial_n u^i \quad \text{on } \partial D \\ \lim_{r\to\infty} r^{1/2} \left(\partial_r u_H^s - iku_H^s\right) = 0 \\ \lim_{r\to\infty} r^{1/2} \left(\partial_r u_M^s - iku_M^s\right) = 0, \quad r = |x| \end{cases} \tag{2}$$


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# Biharmonic Wave Decomposition

We can reformulate the scattering problem (1) as

$$\begin{cases} \Delta u_H^s + k^2 u_H^s = 0, \quad \Delta u_M^s - k^2 u_M^s = 0 \quad \text{in } \mathbb{R}^2 \setminus \overline{D} \\ u_H^s + u_M^s = -u^i, \quad \partial_n u_H^s + \partial_n u_M^s = -\partial_n u^i \quad \text{on } \partial D \\ \lim\limits_{r\to\infty} r^{1/2} \left(\partial_r u_H^s - iku_H^s\right) = 0 \\ \lim\limits_{r\to\infty} r^{1/2} \left(\partial_r u_M^s - iku_M^s\right) = 0, \quad r = |x| \end{cases} \tag{2}$$

## Remark (Exponential Decay of $u_M^s$)

*The evanescent parts $u_M^s$ and $\partial_n u_M^s$ exhibit exponential decay as $r = |x| \to \infty$ for the fixed wavenumber $k$ as $kr \to \infty$. Specifically, $u_M^s$ satisfies*

$$u_M^s(x) = O\left(\frac{e^{-kr}}{r^{\frac{1}{2}}}\right), \; r \to \infty$$



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# Far-Field Pattern of the Biharmonic Scattered Field

Because of the exponential decay of the evanescent part $u_M^s$ and $\partial_n u_M^s$, it follows from the biharmonic wave decomposition that the far-field patterns of $u^s$ and its propagative part $u_H^s$ coincide up to a constant depending on $k$, i.e.,

$$u^\infty(\hat{x}) = C(k)\, u_H^\infty(\hat{x}),$$

where $C(k) = -1/2k^2$. The far-field operator $\mathcal{F} : L^2(\mathbb{S}^1) \to L^2(\mathbb{S}^1)$ can be equivalently defined as

$$(\mathcal{F}g)(\hat{x}) = \int_{\mathbb{S}^1} C(k)\, u_H^\infty(\hat{x}, d; k) g(d)\, ds(d)$$

**Problem (Inverse Cavity Scattering Problem)**

*Given $\mathcal{F}$ for a range of wave numbers obtain qualitative information about the cavity $D$ in a thin elastic plate.*



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# Part 3: Direct Imaging Methods



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# Reconstruction Methods

1. Iterative methods to determine $D$ (expensive optimization; a good initial guess is needed; only one or a few incident waves are needed; reconstructions are reasonably good)
2. Domain decomposition methods (solve an ill-posed linear integral equation first to reduce computational expense, then optimize)
3. **Direct imaging methods** (avoid optimization entirely, solve many ill-posed integral equations, requires a lot of multistatic data but **no a priori information**; partial qualitative information about the scatterer is obtained)



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# Reconstruction of $D$ via Direct Imaging Methods

## Remark (Shape Reconstruction)

**Direct Imaging Methods**: *the idea is to construct an indicator test function* $I$ *that will test whether a point* $z$ *lies inside or outside the scatterer.*

**Benefits:** *can reconstruct the shape of the scatterer in a computational simple manner with* **no a priori information**.



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# Reconstruction of $D$ via Direct Imaging Methods

**Remark (Shape Reconstruction)**

**Direct Imaging Methods**: *the idea is to construct an indicator test function $I$ that will test whether a point $z$ lies inside or outside the scatterer.*
**Benefits:** *can reconstruct the shape of the scatterer in a computational simple manner with* **no a priori information**.

- Assume only the location and shape of the object is needed (e.g., looking for a crack or cavity).
- Based on model, derive an indicator test function $I(z)$, depending on coordinates, so that

$$I(z) = \begin{cases} 0, & z \notin \text{ object} \\ 1, & z \in \text{ object} \end{cases}$$

- $I(z)$ must be fast to compute from the scattered or far-field data.



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# Reconstruction of $D$ via Direct Imaging Methods

Direct methods for the solutions; no need for iterative computations

- **Colton-Kirsch** Linear Sampling Method published in 1996
- **Ikehata** Probe Method published in 1998
- **Kirsch** Factorization Method published 1998
- **Ikehata** Enclosure Method published 1999
- **Potthast** Singular Sources Method published 2000
- **Potthast & Luke** No Response Test published 2003
- **Potthast** Orthogonality Sampling Method published 2010
- **Liu** Direct Sampling Method published 2016



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# Sampling Methods

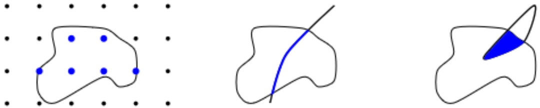

Figure: Types of Sampling

- ‘96 Colton – Kirsch: linear sampling method, factorization (point sampling in grid)
- ‘98 Ikehata: probing method (curve); ‘00 Potthast: singular source method (curve/needle)
- … Luke, Potthast, Sylvester, Kusiak, Ikehata: range test, no response test, enclosure method (sets/planes)



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# Probe Method

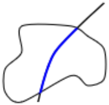

Figure: Probing the scatterer with curve/needle

The probe method (Ikehata '98) is a method of **probing inside** the given material by using the sequence of the energy gap

$$I_n := \langle (\Lambda_0 - \Lambda_D)(v_n|_{\partial\Omega}), \overline{v_n}|_{\partial\Omega} \rangle$$

for a specially chosen sequence $\{v_n\}$ of solutions of the governing equation for the background scatterer/cavity, with $D \subset \text{int}(\Omega)$.

- $I_n \to \infty$ on a given curve
- $I_n$ is convergent outside the curve



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# Singular Source Method

Like the probe method, the SSM (Potthast '00) is a method of **probing inside** the given material but now using the **magnitude of the scattered field of singular sources**

$$I(z) := |\Psi^s(z,z)|.$$

Approximated by backprojection of the form

$$\Psi^s(y,z) \approx \int_{\mathbb{S}^{n-1}} \int_{\mathbb{S}^{n-1}} u^\infty(\hat{x},d) g(\hat{x},y) g(-d,z)\, ds(d) ds(\hat{x})$$

for explicitly constructed kernels $g(\cdot,\cdot)$.

- $I(z) \to \infty$ on a given curve (as $z \to \partial D$)
- $I(z)$ is convergent outside the curve

# Enclosure Method

Figure: Intersecting the scatterer with sets

The enclosure method (Ikehata '99) enables one to construct the support of unknown convex polygons from the knowledge of one measured field.

$$v = e^{\tau x \cdot (\omega + i\omega^{\perp})}$$

is a special harmonic incident field.

- $\Omega$ is some domain known to contain the unknown scatterer
- $D \subset \text{int}(\Omega)$

# Enclosure Method

At the corners of polygonal scatterers, the following indicator function becomes unbounded

$$I_\omega(\tau, t) := e^{-\tau t} \left\{ \left\langle \left.\frac{\partial u}{\partial n}\right|_{\partial\Omega}, v|_{\partial\Omega} \right\rangle - \left\langle \left.\frac{\partial v}{\partial n}\right|_{\partial\Omega}, u|_{\partial\Omega} \right\rangle \right\}$$

with $\tau > 0$, $t \in \mathbb{R}$, $u$ the unknown, $\omega \in \mathbb{S}^{n-1}$ the direction vector.



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# Enclosure Method

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with $\tau > 0$, $t \in \mathbb{R}$, $u$ the unknown, $\omega \in \mathbb{S}^{n-1}$ the direction vector.

- Benefit: requires only one special harmonic incident field
- Benefit: so doesn't require too much data; works well with limited aperture data
- Drawback: only works for convex polygonal scatterers



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# Factorization Method

Most direct imaging/sampling methods give only sufficient conditions for $z \in \text{supp } D$. Linear sampling method is no exception. But factorization method (Kirsch 90's, Grinberg 00's) gives necessary & sufficient conditions, assuming additional assumptions.

Idea

$$u^i(x) = \int_{\mathbb{S}^{n-1}} e^{ikx\cdot d} g(d)\, ds(d), \quad g \in L^2(\mathbb{S}^{n-1})$$

$$u^s(x) = \frac{e^{ik|x|}}{|x|^{(n-1)/2}} u^\infty(\hat{x}) + O\left(\frac{1}{|x|^{n-2}}\right)$$

the far-field operator

$$\mathcal{F} : L^2(\mathbb{S}^{n-1}) \to L^2(\mathbb{S}^{n-1}), \quad \mathcal{F}g = A_g$$

is factored as

$$\mathcal{F} = -\mathcal{G}\mathcal{T}\mathcal{G}^*, \quad \mathcal{G} \text{ compact, } \mathcal{T} \text{ isomorphism}$$



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$$\mathcal{F} : L^2(\mathbb{S}^{n-1}) \to L^2(\mathbb{S}^{n-1}), \quad \mathcal{F}g = A_g$$

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Range of $\mathcal{G}$ can be characterized and gives information about supp$(D)$. But the main benefit is that if

- $\mathcal{T}$ is strictly coercive
- $\mathcal{F}$ is a normal compact operator (so it has a 'positive square root')

then Range$(\mathcal{G})$ = Range$(|\mathcal{F}|^{1/2})$.
Range of $\mathcal{F}$ can be directly characterized under these assumptions, giving direct info on supp$(D)$.



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# Part 4: Reconstruction of the Cavity $D$ via the Linear Sampling Method



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# Uniqueness Result

**Theorem (P. Li & H. Dong, 2023)**

*Let $D_1$ and $D_2$ be two cavities meeting the clamped boundary conditions, with corresponding far-field patterns $u_1^\infty$ and $u_2^\infty$ satisfying*

$$u_1^\infty(\hat{x}, d) = u_2^\infty(\hat{x}, d), \quad \forall \hat{x}, d \in \mathbb{S}^1.$$

*Then $D_1 = D_2$.*

- This result guarantees uniqueness of the inverse cavity scattering problem with clamped boundary conditions.
- Proof of the result is based on the reciprocity relations of the far-field patterns of the corresponding propagative and evanescent parts.



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# The Far-Field Equation

$G_H(x,z) := \frac{i}{4} H_0^{(1)}(k|x-z|)$, $x \neq z$: the fundamental solution of the Helmholtz equation.
$G_M(x,z) := \frac{i}{4} H_0^{(1)}(ik|x-z|)$, $x \neq z$: the fundamental solution to the modified Helmholtz. Then

$$G(x,y) = \frac{1}{2k^2}(G_M(x,y) - G_H(x,y)), \quad x \neq y$$

is the fundamental solution of $\Delta^2 - k^4$. $G$ has the far-field pattern

$$G^\infty(\hat{x}) = -\frac{1}{2k^2}\frac{e^{i\pi/4}}{\sqrt{8\pi k}} e^{-ikz\cdot\hat{x}}.$$



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# The Far-Field Equation

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$$G^\infty(\hat{x}) = -\frac{1}{2k^2}\frac{e^{i\pi/4}}{\sqrt{8\pi k}}e^{-ikz\cdot\hat{x}}.$$

$$(\mathcal{F}g_z)(\hat{x}) = G^\infty(\hat{x},z), \quad g_z \in L^2(\mathbb{S}^1),\ z \in \mathbb{R}^2 \qquad (3)$$



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# On Solving the Far-Field Equation

$$(\mathcal{F}g_z)(\hat{x}) = G^\infty(\hat{x}, z) \quad g_z \in L^2(\mathbb{S}^1),\ z \in \mathbb{R}^2$$

Let $z \in D$ and suppose that $g_z$ solves the far-field equation.

- Rellich's Lemma $\implies u^s(x) = G(x, z)$ in $\mathbb{R}^2 \setminus \overline{D}$
- $-\begin{pmatrix} v_g \\ \partial_n v_g \end{pmatrix} = \begin{pmatrix} G(x, z) \\ \partial_n G(x, z) \end{pmatrix}$ on $\partial D$
- As $z \in D \to \partial D$, $\begin{pmatrix} G(x, z) \\ \partial_n G(x, z) \end{pmatrix} \to \infty$ and so does $\begin{pmatrix} v_g \\ \partial_n v_g \end{pmatrix} \to ||g||_{L^2} \to \infty$.
- $v_g(x) := \int_{\mathbb{S}^1} g(d) e^{ikx \cdot d}\, ds(d)$ is the Herglotz wave function.



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# On Solving the Far-Field Equation

In general, the far-field equation does not have a solution for any $z \in \mathbb{R}^2$ since $\mathcal{F}$ is compact.

For $z \in D$, the far-field equation has a solution if and only if the **interior boundary value problem**

$$\Delta^2 w_z - k^4 w_z = 0 \quad \text{in } D$$
$$w_z + G(\cdot, z) = 0, \quad \partial_n w_z + \partial_n G(\cdot, z) = 0 \text{ on } \partial D$$

has a solution $w_z$ such that $w_z = v_g$ is a Herglotz function with kernel $g$ on $\partial D$.

- Equivalently, this holds if $k^4 \neq$ Dirichlet eigenvalue of $-\Delta^2$ in $D$.



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# Factorization of the Far-Field Operator $\mathcal{F}$

Define the following operators

$$\mathcal{G} : H^{3/2}(\partial D) \times H^{1/2}(\partial D) \to L^2(\mathbb{S}^1) \; : \; \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \mapsto w^\infty$$

$$\mathcal{H} : L^2(\mathbb{S}^1) \to H^{3/2}(\partial D) \times H^{1/2}(\partial D) \; : \; g \mapsto \begin{pmatrix} v_g \\ \partial_n v_g \end{pmatrix}$$

Then

$$\boxed{\mathcal{F} = -\mathcal{G}\mathcal{H}}$$

- $\mathcal{G}$ maps boundary data of the exterior boundary value problem to the far-field pattern of the solution $w$ to the exterior problem
- $\mathcal{H}$ is the Herglotz wave operator



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# Range Characterization of the Cavity $D$

The **linear sampling method** is a direct imaging method based on the following range characterization of the cavity $D$:

**Lemma**

*$z \in D$ if and only if $G^{\infty}(\hat{x}, z) \in$ Range$(\mathcal{G})$.*

This result helps justify the use of the indicator test function

$$I(z) := \frac{1}{||g_z||_{L^2(\mathbb{S}^1)}}$$

LSM states

- $I(z) > 0$ if $z \in D$
- $I(z) \to 0$ as $z \to \partial D$ and $I(z) = 0$ if $z \notin \partial D$



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# Reconstruction of the Cavity $D$ via the LSM

## Theorem (The Linear Sampling Method)

- *Suppose* $z \in D$. *Given* $\epsilon > 0$ *there exists a regularized solution* $g_{z,\epsilon} \in L^2(\mathbb{S}^1)$ *to the far-field equation such that*

$$||\mathcal{F} g_{z,\epsilon} - G^\infty(\cdot, z)||_{L^2(\mathbb{S}^1)} < \epsilon.$$

  *Furthermore,* $||g_{z,\epsilon}||_{L^2(\mathbb{S}^1)}$ *is* **unbounded** *as* $z \to z^* \in \partial D$.
- *Suppose* $z \notin D$. *Then the regularized solution of the far-field equation* $g_{z,\epsilon}$ *satisfies*

  $||g_{z,\epsilon}||_{L^2(\mathbb{S}^1)}$ *is* **unbounded** *as* $\epsilon \to 0$, *assuming that*

$$||\mathcal{F} g_{z,\epsilon} - G^\infty(\cdot, z)||_{L^2(\mathbb{S}^1)} \to 0 \quad \text{as } \epsilon \to 0.$$

# Reconstruction of the Cavity $D$ via the LSM

- Construct a grid $\mathcal{G}$
- For each $z_i \in \mathcal{G}$, solve the regularized far-field equation $(\alpha I + \mathcal{F}^*\mathcal{F})g_{z_i} = \mathcal{F}^* G^\infty(\hat{x}, z_i)$
- To reconstruct $\partial D$, we plot $z_i \mapsto 1/||g_{z_i,\epsilon}||_{L^2(S^1)}$ for each point $z_i$ in some grid point in $\mathbb{R}^2$.

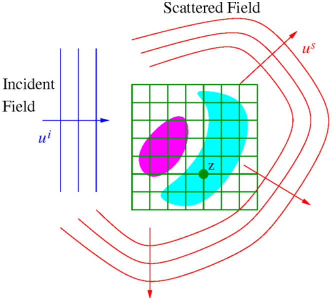


Figure: Shape Reconstruction via Sampling in a Grid

# Part 5: Ongoing Future Work

# Ongoing Future Work

- numerical implementation of the linear sampling method with far-field data
- incorporate the presence of noisy data in implementation
- other boundary conditions (e.g., free plate, simply supported)
- formulate the factorization method for the inverse cavity scattering problem based on the symmetric factorization of $\mathcal{F}$



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