

Name: _____ Date: _____

Instructions. Show all work with clear, logical steps. No work or hard-to-follow work will lose points. Scientific calculators are allowed. **2 points for name/date**

Problem 1. (4 points each) Evaluate the double integral

$$\int_{-1}^2 \int_y^2 (x + y) dx dy.$$

Solution.

Compute the inner integral:

$$\int_y^2 (x + y) dx = \left[\frac{1}{2}x^2 + yx \right]_{x=y}^{x=2} = \left(\frac{1}{2} \cdot 4 + 2y \right) - \left(\frac{1}{2}y^2 + y^2 \right) = 2 + 2y - \frac{3}{2}y^2.$$

Now integrate with respect to y :

$$\int_{-1}^2 \left(2 + 2y - \frac{3}{2}y^2 \right) dy = \left[2y + y^2 - \frac{1}{2}y^3 \right]_{-1}^2.$$

Evaluate:

$$(4 + 4 - 4) - \left(-2 + 1 + \frac{1}{2} \right) = 4 - \left(-\frac{1}{2} \right) = \frac{9}{2}.$$

$$\boxed{\frac{9}{2}}$$

Problem 2. (4 points) Set up the limits of the double integral

$$\iint_R \frac{1}{x^2 + y^2} dA,$$

in terms of $dy dx$, where R is the region bounded by $y = 5x$, the x -axis, and $x = 2$. **Do not evaluate.**

Solution.

The region is bounded by: - below: $y = 0$, - above: $y = 5x$, - left: $x = 0$, - right: $x = 2$.
Thus the integral in the order $dy dx$ is

$$\boxed{\int_0^2 \int_0^{5x} \frac{1}{x^2 + y^2} dy dx}$$