MA 16020 — Quiz 6 **Wednesday, October 29, 2025**

Name: _____ Date: _____

Instructions. Show all work with clear, logical steps. No work or hard-to-follow work will lose points. Scientific calculators are allowed. **2** points for name/date

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Problem 1. (4 points) What is the **integrating factor** of the following differential equation? Be sure to simplify.

$$2y' + \left(\frac{6}{x}\right) y = 10 \ln x$$

Problem 2. (4 points) Use separation of variables to find the general solution to the following differential equation.

$$\frac{dy}{dt} = 8e^{-4t-y}.$$

Solutions — Quiz 6

Problem 1. We are asked to find the integrating factor for

$$2y' + \frac{6}{x}y = 10\ln x.$$

Step 1: Write in standard linear form $y' + p(x)y = q(x)$.

$$y' + \frac{3}{x}y = 5\ln x.$$

Step 2: Identify $p(x) = \frac{3}{x}$.
Step 3: Compute the integrating factor:

$$I(x) = e^{\int p(x)\,dx} = e^{\int \frac{3}{x}\,dx} = e^{3\ln|x|} = |x|^3.$$

Assuming $x > 0$:

$$\boxed{I(x) = x^3.}$$

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Problem 2. We are asked to solve

$$\frac{dy}{dt} = 8e^{-4t-y}.$$

Step 1: Separate the variables.

$$e^y\,dy = 8e^{-4t}\,dt.$$

Step 2: Integrate both sides.

$$\int e^y\,dy = \int 8e^{-4t}\,dt$$

$$e^y = -2e^{-4t} + C.$$

Step 3: Solve for y.

$$e^y = C - 2e^{-4t} \quad \Rightarrow \quad y = \ln\bigl(C - 2e^{-4t}\bigr).$$

$$\boxed{y = \ln\bigl(C - 2e^{-4t}\bigr)}$$

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Final Answers:

$$\boxed{I(x) = x^3, \qquad y = \ln(C - 2e^{-4t}).}$$