MA 16020 — Quiz 5 Solutions **Wednesday, October 7, 2025**

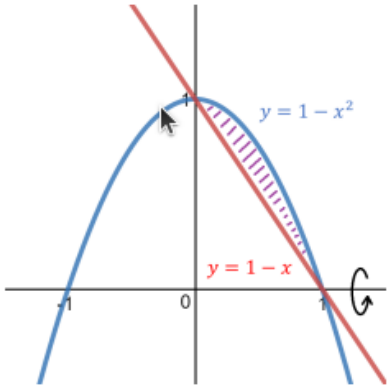
Name: _____ Date: _____

Problem 1

Given:

$$y = 1 - x^2, \quad y = 1 - x$$

Setup the integral that finds the volume of the solid formed by revolving the region about the **x-axis** using the washer method.



Step 1: Find intersection points

Set $1 - x^2 = 1 - x \Rightarrow x^2 - x = 0 \Rightarrow x(x-1) = 0 \Rightarrow x = 0, 1$

Step 2: Set up integral

$$V = \pi \int_0^1 \left[(1-x^2)^2 - (1-x)^2\right] dx$$

—

Optional Evaluation for the Curious

$$(1-x^2)^2 - (1-x)^2 = (1 - 2x^2 + x^4) - (1 - 2x + x^2) = -2x^2 + x^4 + 2x - x^2 = x^4 - 3x^2 + 2x$$

$$V = \pi \int_0^1 (x^4 - 3x^2 + 2x)dx = \pi \left[\frac{x^5}{5} - x^3 + x^2\right]_0^1 = \pi \left(\frac{1}{5} - 1 + 1\right) = \frac{\pi}{5}$$

$$\boxed{V = \frac{\pi}{5}}$$

—

Problem 2

Given: $y = x + 1$, $x = 0$, $y = 3$ revolve about the **y-axis** using disk method.

—

Step 1: Solve for x in terms of y

$$y = x + 1 \Rightarrow x = y - 1$$

- Limits of integration: $y \in [1, 3]$
- Disk radius: distance from y-axis $= x = y - 1$
- Thickness: dy

—

Step 2: Set up integral

$$V = \pi \int_1^3 (y-1)^2 dy$$

—

Step 3: Evaluate

$$\int_1^3 (y-1)^2 dy = \int_0^2 u^2 du \quad (u = y - 1) = \left. \frac{u^3}{3} \right|_0^2 = \frac{8}{3}$$

$$V = \pi \cdot \frac{8}{3} = \frac{8\pi}{3}$$

$$\boxed{V = \frac{8\pi}{3}}$$