MA 16020 — Quiz 4 Solutions **Wednesday, October 1, 2025**

Name: ______________________________ Date: ____________

Instructions. Show all work with clear, logical steps. No work or hard-to-follow work will lose points. Scientific calculators are allowed.

Problem 1. (4 points) Determine if the following integral is convergent or divergent. If it is convergent, find its value.

$$\int_{-\infty}^{0} \frac{1}{\sqrt{3-x}}\,dx$$

Solution. The integral is improper because the lower limit is $-\infty$. We rewrite it as a limit:

$$\int_{-\infty}^{0} \frac{1}{\sqrt{3-x}}\,dx = \lim_{t\to-\infty}\int_{t}^{0} \frac{1}{\sqrt{3-x}}\,dx.$$

Let $u = 3 - x \implies du = -dx$, so $dx = -du$. When $x = t$, $u = 3 - t$, and when $x = 0$, $u = 3$. Then

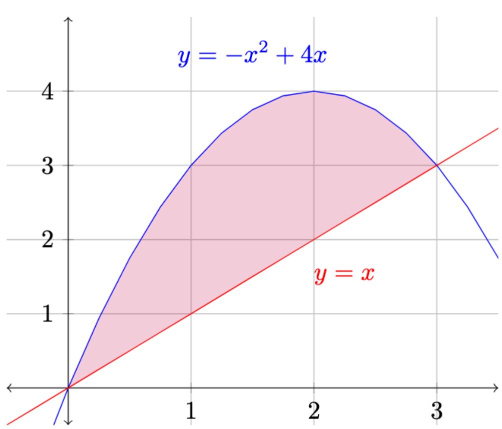
$$\int_{t}^{0} \frac{1}{\sqrt{3-x}}\,dx = \int_{3-t}^{3} \frac{1}{\sqrt{u}}(-du) = \int_{3}^{3-t} \frac{1}{\sqrt{u}}\,du = 2\left(\sqrt{3-t}-\sqrt{3}\right).$$

Taking the limit as $a \to -\infty$:

$$\lim_{t\to-\infty} 2\left(\sqrt{3-t}-\sqrt{3}\right) = \infty.$$

Therefore, the integral **diverges**.

Problem 2. Set up the definite integral that computes the area shown below with respect to x. (4 points)



Solution. The curves are $y = -x^2 + 4x$ (top) and $y = x$ (bottom). First, find the intersection points:

$$-x^2 + 4x = x \implies -x^2 + 3x = 0 \implies x = 0, 3.$$

So the area is given by

$$\text{Area} = \int_0^3 \big((-x^2 + 4x) - x\big)\, dx = \int_0^3 (-x^2 + 3x)\, dx.$$