
Problem 1. (4 points)

Find the partial fraction decomposition of $\dfrac{108}{x^2 - 81}$.

(NO NEED TO EVALUATE THE INTEGRAL!)

Solution: Factor the denominator:

$$x^2 - 81 = (x - 9)(x + 9).$$

So we set

$$\frac{108}{x^2 - 81} = \frac{A}{x - 9} + \frac{B}{x + 9}.$$

Multiply through by $(x - 9)(x + 9)$:

$$108 = A(x + 9) + B(x - 9).$$

Method 1 (expand and compare coefficients):

$$108 = (A + B)x + (9A - 9B).$$

So,

$$A + B = 0, \qquad 9A - 9B = 108.$$

From $A + B = 0$, we get $B = -A$. Substituting:

$$18A = 108 \quad \Rightarrow \quad A = 6,\ B = -6.$$

Method 2 (cover-up trick): Plugging in $x = 9$ eliminates B:

$$108 = A(9 + 9) \quad \Rightarrow \quad A = 6.$$

Plugging in $x = -9$ eliminates A:

$$108 = B(-9 - 9) \quad \Rightarrow \quad B = -6.$$

Therefore,

$$\frac{108}{x^2 - 81} = \frac{6}{x - 9} - \frac{6}{x + 9}.$$

Problem 2. (4 points) Use partial fractions to evaluate the integral

$$\int \frac{5x + 16}{x^2 + 8x}\, dx.$$

Solution: Factor the denominator:

$$x^2 + 8x = x(x+8).$$

So we write

$$\frac{5x+16}{x(x+8)} = \frac{A}{x} + \frac{B}{x+8}.$$

Multiply through by $x(x+8)$:

$$5x + 16 = A(x+8) + Bx.$$

Expanding,

$$5x + 16 = (A+B)x + 8A.$$

Thus,

$$A + B = 5, \qquad 8A = 16.$$

So $A = 2$, and then $B = 3$.

$$\frac{5x+16}{x(x+8)} = \frac{2}{x} + \frac{3}{x+8}.$$

Now integrate:

$$\int \frac{5x+16}{x^2+8x}\, dx = \int \left(\frac{2}{x} + \frac{3}{x+8} \right) dx = 2 \ln |x| + 3 \ln |x+8| + C.$$

End of Solutions.