MA 16020 — Quiz 2 Solutions **Wednesday, September 10, 2025**

Name: ______________________________ Date: ____________

Solutions. Show all work with clear, logical steps. No work or hard-to-follow work will lose points. Scientific calculators are allowed.

Problem 1. (4 points)

Use u-substitution to evaluate the following integral:

$$\int_1^2 x\sqrt{x-1}\,dx$$

Solution: Use the substitution $u = x - 1$, so $du = dx$ and $x = u + 1$. Change the limits:

$$x = 1 \Rightarrow u = 0, \quad x = 2 \Rightarrow u = 1$$

The integral becomes:

$$\int_0^1 (u+1)\sqrt{u}\,du = \int_0^1 (u^{3/2} + u^{1/2})\,du$$

Integrate term by term:

$$\int_0^1 u^{3/2} du = \frac{2}{5}, \quad \int_0^1 u^{1/2} du = \frac{2}{3}$$

Add the results:

$$\frac{2}{5} + \frac{2}{3} = \frac{6+10}{15} = \frac{16}{15}$$

$$\boxed{\int_1^2 x\sqrt{x-1}\,dx = \frac{16}{15}}$$

Problem 2. (4 points)

Set up an integral and find the average value of

$$f(x) = xe^{x^2}, \quad 0 < x < 2$$

Solution: The average value formula is:

$$f_{\text{avg}} = \frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{2-0}\int_0^2 xe^{x^2}\,dx$$

Use the substitution $u = x^2 \Rightarrow du = 2xdx$, so $xdx = \frac{du}{2}$ and note

$$u(0) = 0^2 = 0, \quad u(2) = 2^2 = 4$$

. Evaluate the integral:

$$\int_0^2 xe^{x^2} dx = \int_0^4 \frac{1}{2}e^u du = \frac{1}{2}e^u|_0^4 = \frac{1}{2}e^4 - \frac{1}{2}e^0 = \frac{1}{2}e^4 - \frac{1}{2}$$

Multiply by the factor for the average value:

$$f_{\text{avg}} = \frac{1}{2} \cdot \left(\frac{1}{2} e^4 - \frac{1}{2} \right)$$

so

$$\boxed{f_{\text{avg}} = \frac{1}{4} e^4 - \frac{1}{4}}$$

Remember—writing your name and full date at the top earns 2 free points!