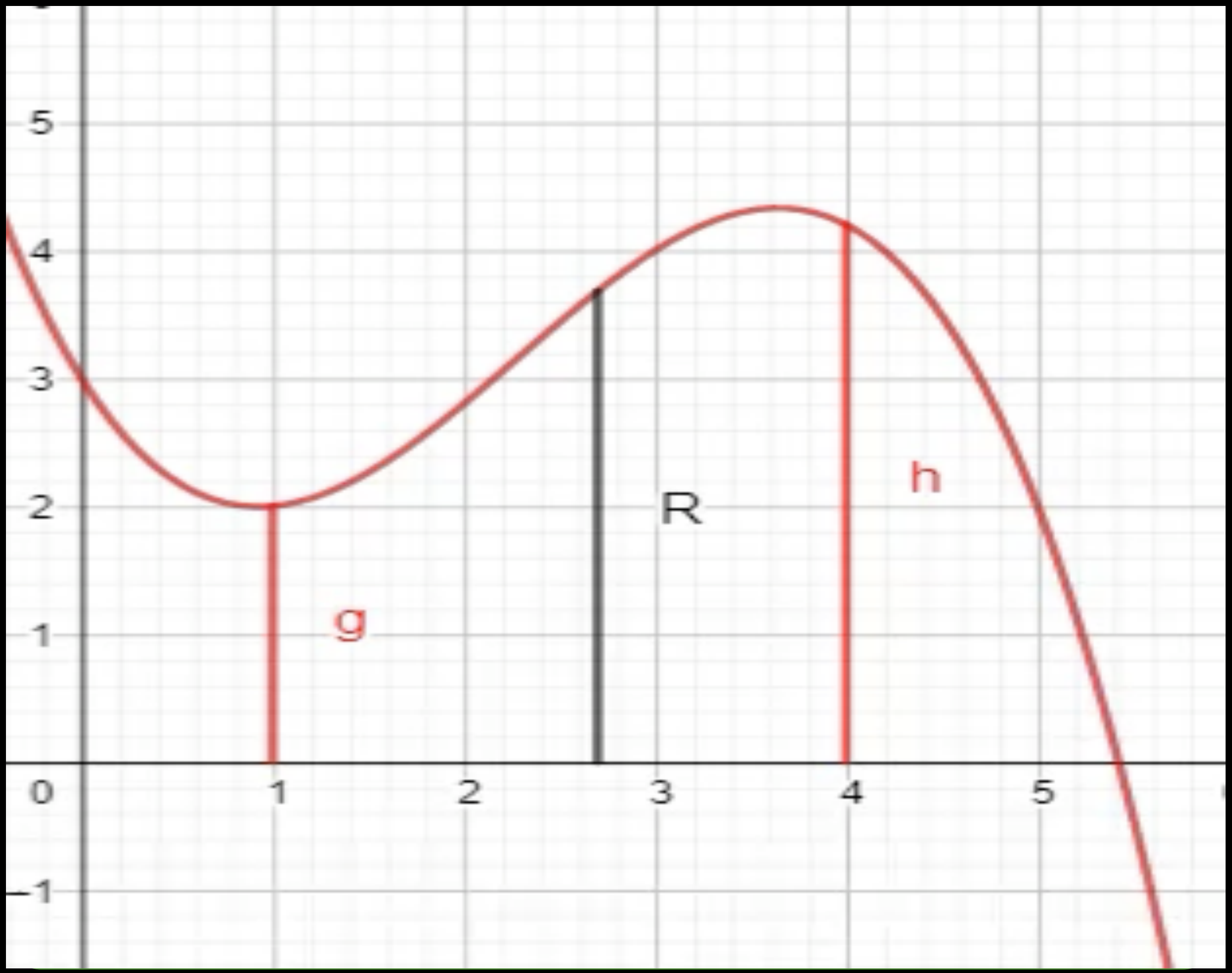
# MA 16020: Lesson 17-18
# Volume By Revolution
# Shell Method
# Pt 1-2

By General Ozochiawaeze

# So far…

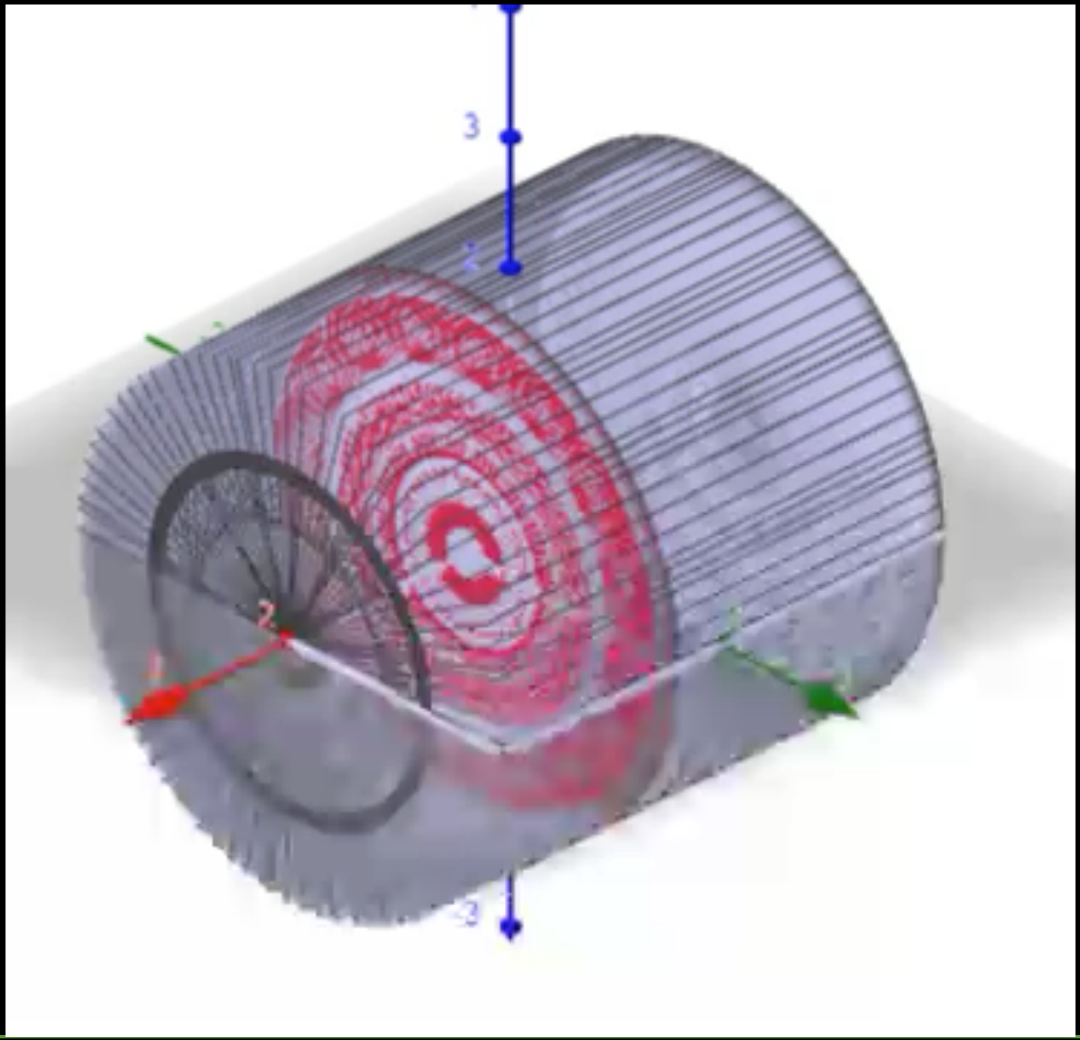
- We have learned how to find the volume of a solid of revolution by integrating
  - In the same way, we calculate the area under a curve
    - Running a line segment of varying length across the region, and adding them up




https://www.geogebra.org/m/tgceabb2#material/tnnhu7gz

# In other words,

- We learned to find the volume of a solid of revolution by
  - Running some area across a shape and add them up.
  - Like in the case of the cylinder shown on the right.



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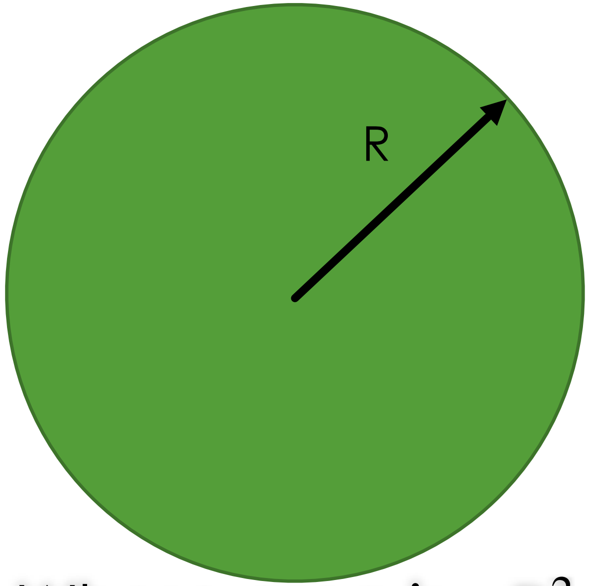
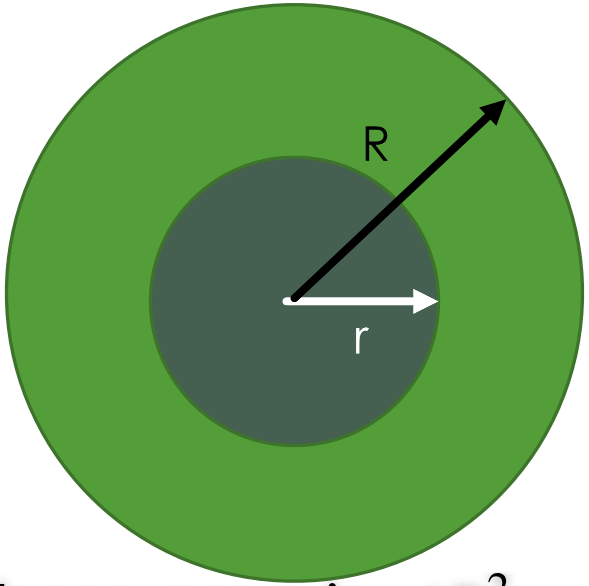
# But what were those "shapes"? ANSWER: CROSS-SECTIONS

Sometimes it was a disk




Whose area is $\pi R^2$

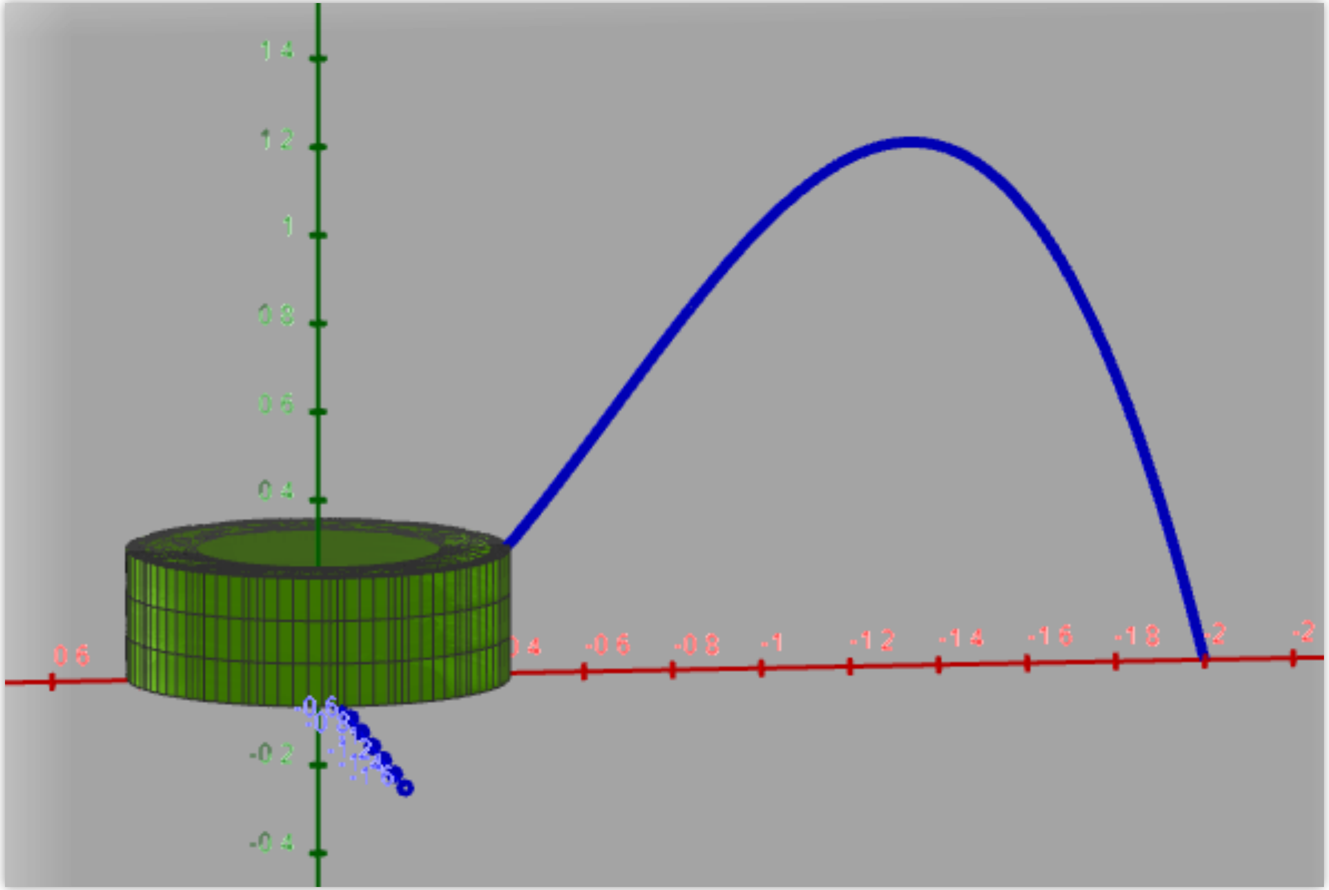
Sometimes it was a washer




Whose area is $\pi(R^2 - r^2)$

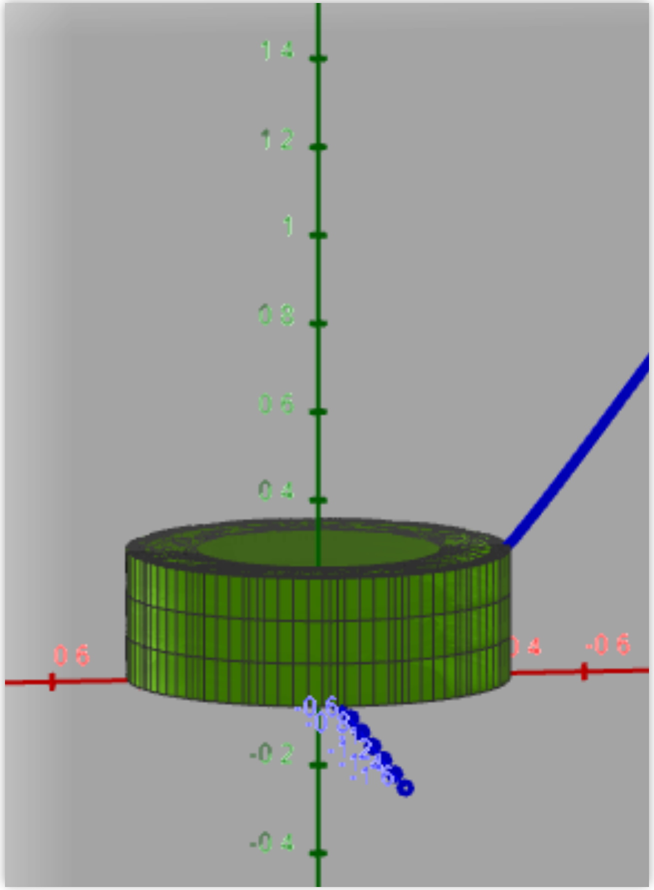
# Cross Section: a (Cylindrical) Shell!

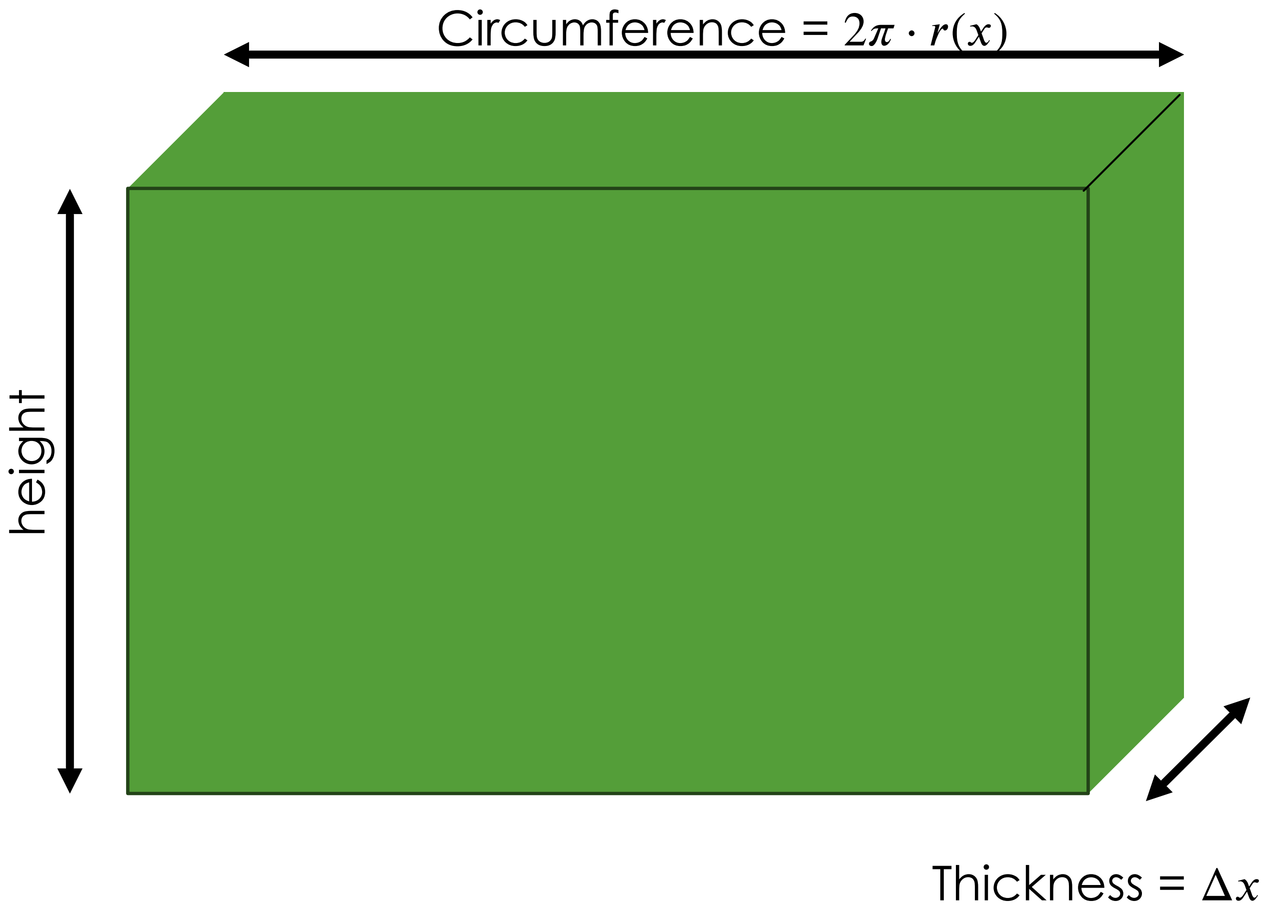
- Before we would find the volume by taking cuts perpendicular to the axis,
  - **In Shells, we take cuts parallel to our axis (as shown in the image in the right)**
- The reason is USEFUL is that
  - For this problem, we no longer have to solve for $x$ in terms of $y$.
- **What's the formula of that shell?**



https://www.geogebra.org/m/jqfyndpu

# Geometry Time: Let's Flatten the Shell …






https://www.geogebra.org/m/jqfyndpu

# Geometry Time: Let's Flatten the Shell …

- So the volume of the **green image** is

$V = \text{circumference} \times \text{height} \times \text{thickness}$

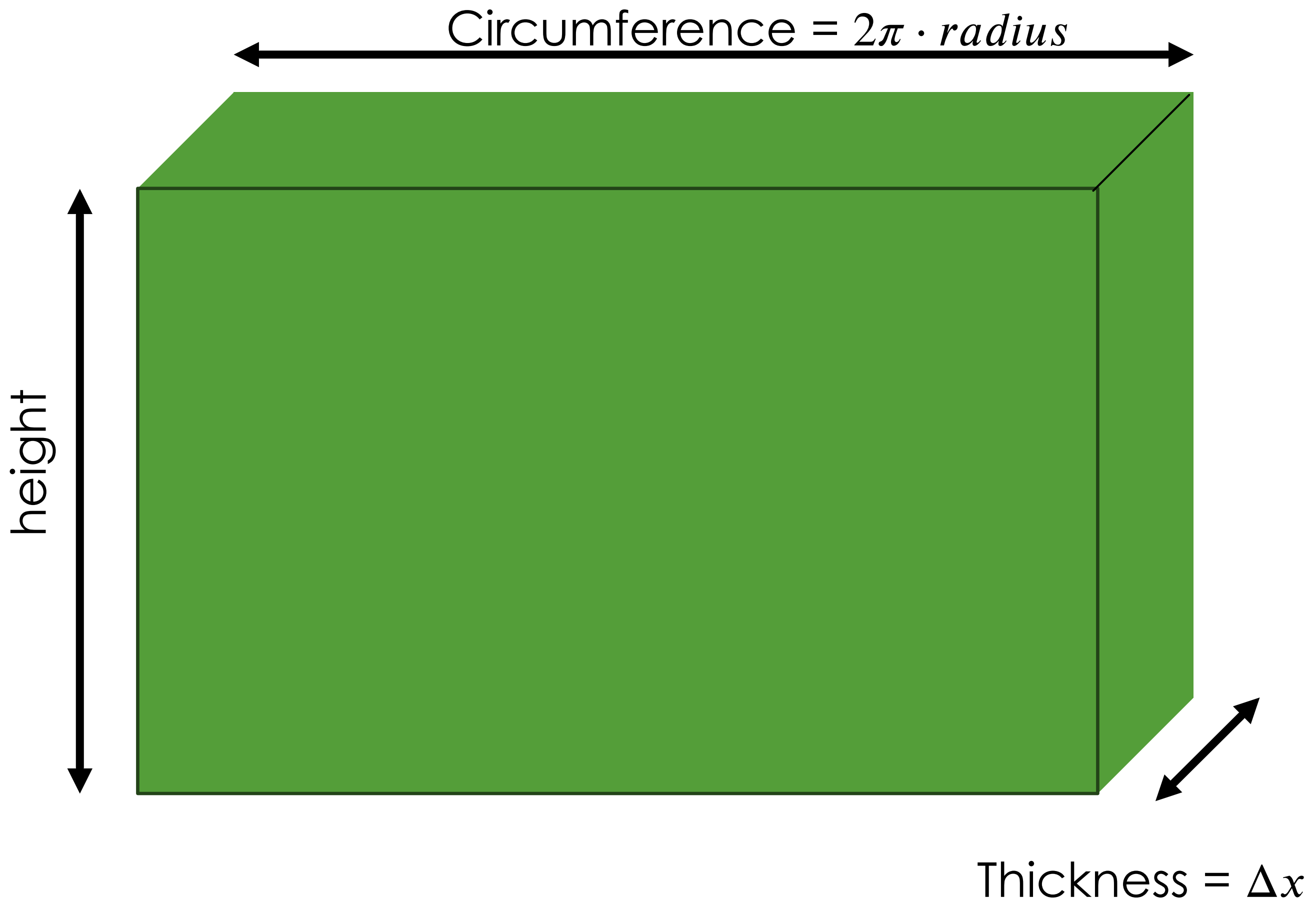
$V = 2\pi \cdot \mathrm{r}(x) \cdot h \cdot \Delta x$

where $r(x)$ is the radius.

- The height is determined if you have one or two functions.
  - i.e. Top - Bottom or Right – Left
- So, in the dx case,

$$V = 2\pi \cdot r(x)\left(Top - Bottom\right) dx \text{ over } [a, b].$$

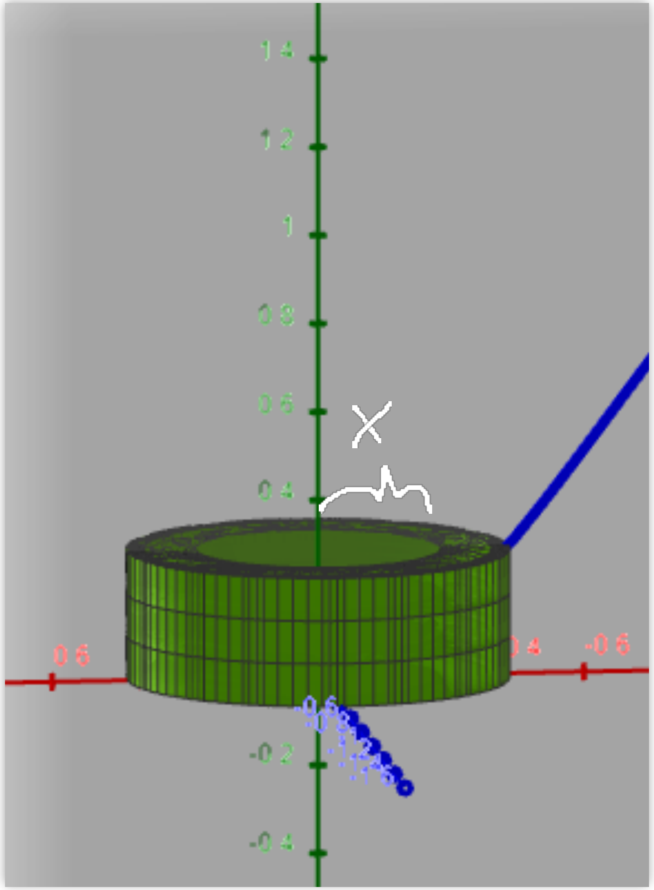
i.e. $V = 2\pi \int_a^b r(x) \cdot (Top - Bottom)dx$



# But what is the radius, $r(x)$?

- To find $r(x)$, we need to find the distance of the shell from the axis of rotation
- So, in the dx case with rotation around the $y$-axis,
  - The shell is $x$ units away from the y-axis. So, $r(x) = x$
- This yields the formula:

$$V = 2\pi \int_a^b x \cdot (Top - Bottom)dx$$

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# One thing about Shell Method Formulas

Since we are just cutting out parallel to the axis, we choose dx or dy in the following way:

- **Rotating around y-axis**

  $\Rightarrow$ **" dx " problem**

$$V = 2\pi \int_a^b x \cdot (Top - Bottom)dx$$

- **Rotating around x-axis**

  $\Rightarrow$ **" dy " problem**

$$V = 2\pi \int_c^d y \cdot \left(Right - Left\right)dy$$

If you need more of an explanation of where the Shell Method comes look at the hidden slides.

Example 1: Find the volume obtained by revolving the region bounded by the curves

$$y = 2x^2 - x^3 \quad \text{and} \quad y = 0$$

About the y-axis.

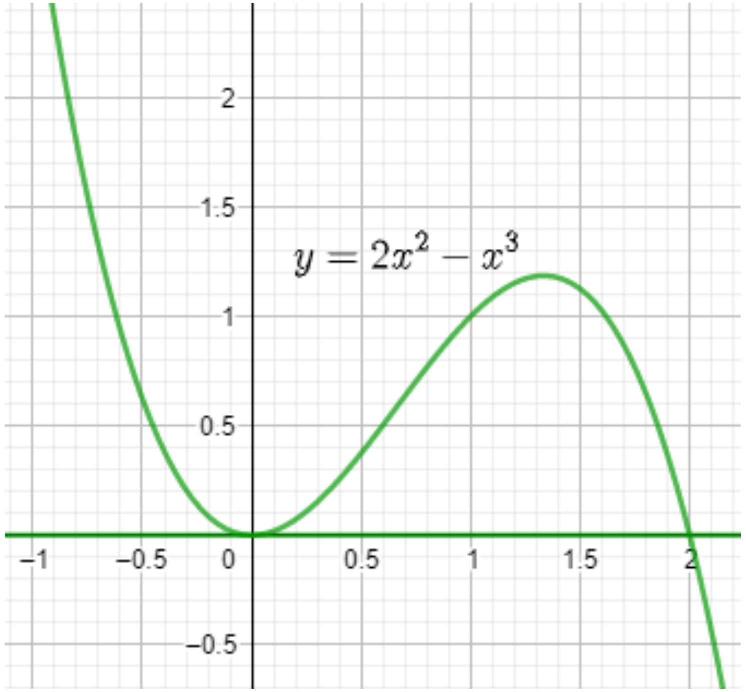
https://www.geogebra.org/m/jqfyndpu

Example 1: Find the volume obtained by revolving the region bounded by the curves

$$y = 2x^2 - x^3 \quad \text{and} \quad y = 0$$

About the y-axis.

Draw the region.

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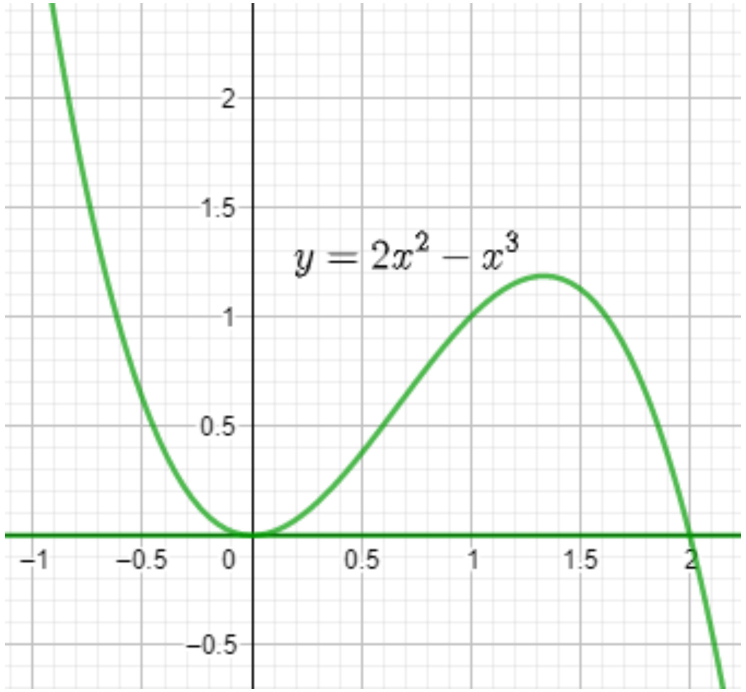
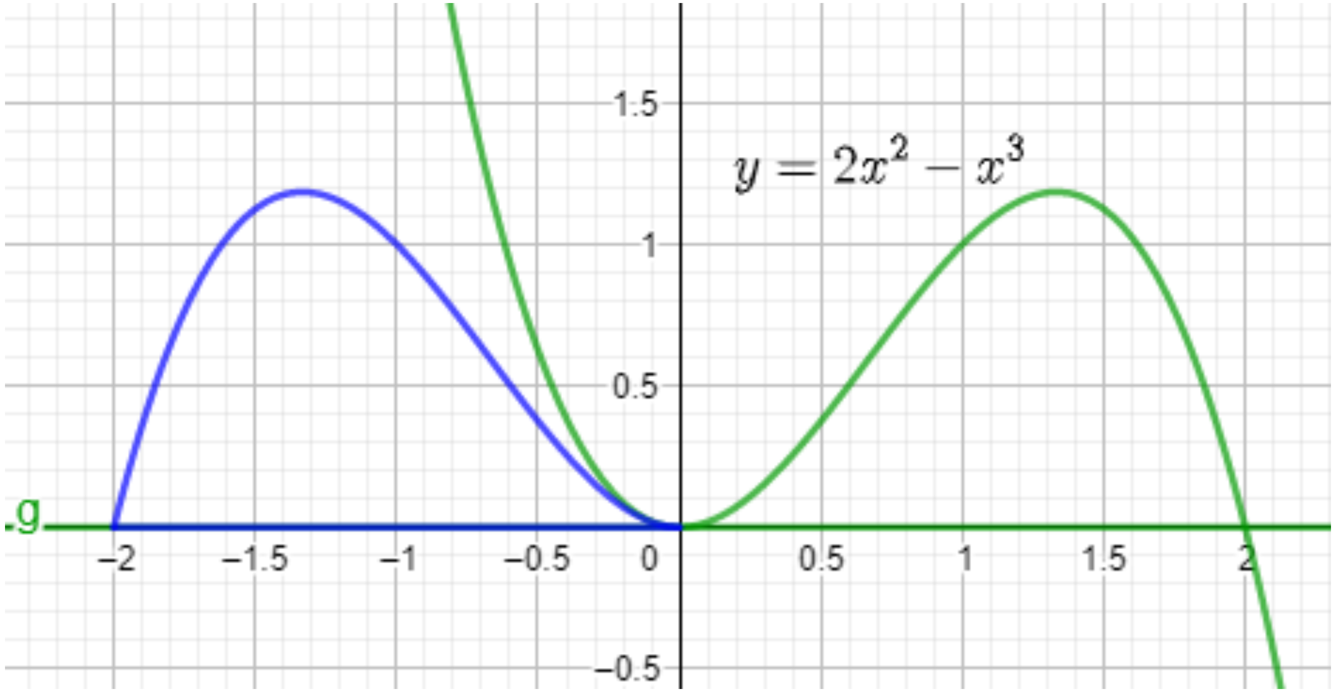
$$y = 2x^2 - x^3 \quad \text{and} \quad y = 0$$

About the y-axis.

Rotation about y-axis

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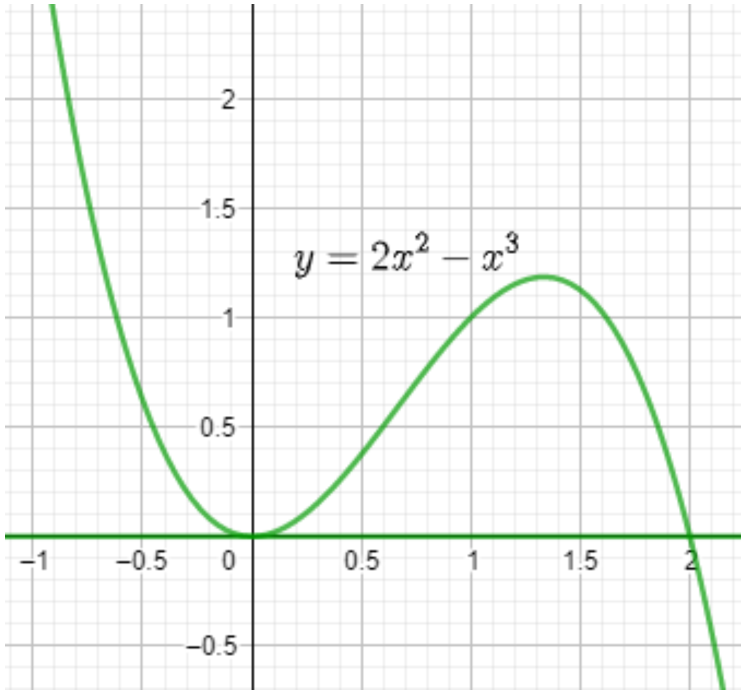
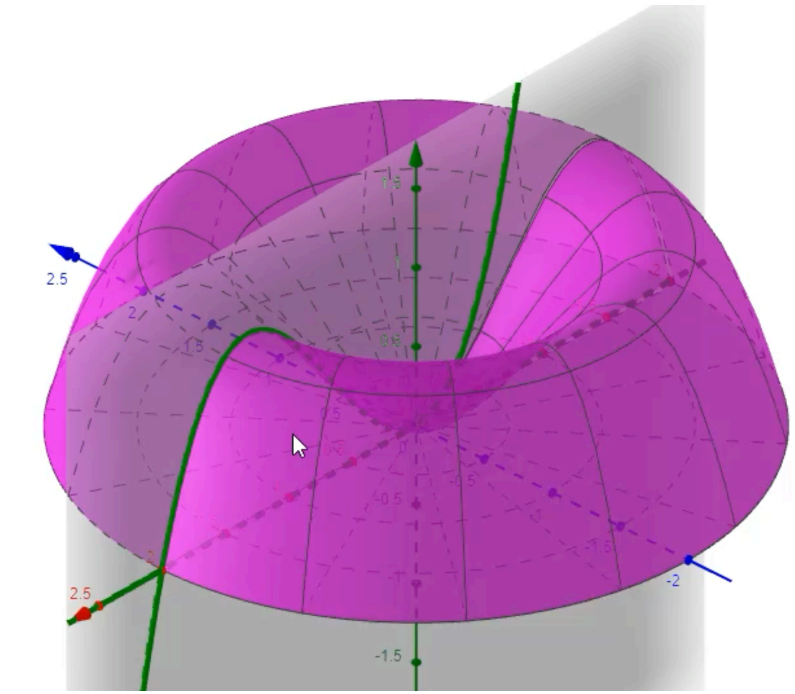
About the y-axis.




**Will use Shell!**

Furthermore, 3-D



https://www.geogebra.org/m/jqfyndpu

Example 2: Find the volume obtained by revolving the region bounded by the curves

$$y = \frac{5}{x^3}, \quad y = 0, \quad x = 1 \quad \text{and} \quad x = 5$$

About the y-axis.

https://www.geogebra.org/m/f3wrypfh#material/aur8qe9f

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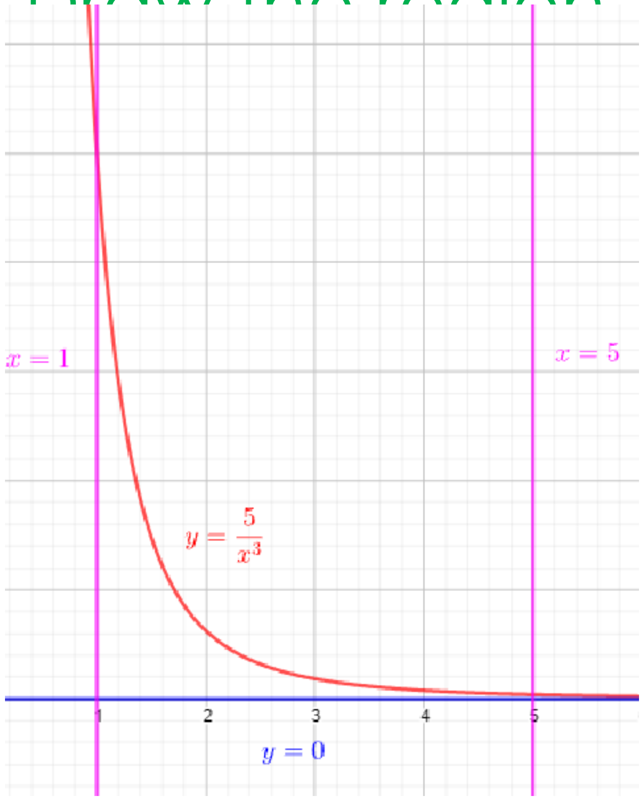
About the y-axis.

Draw the region

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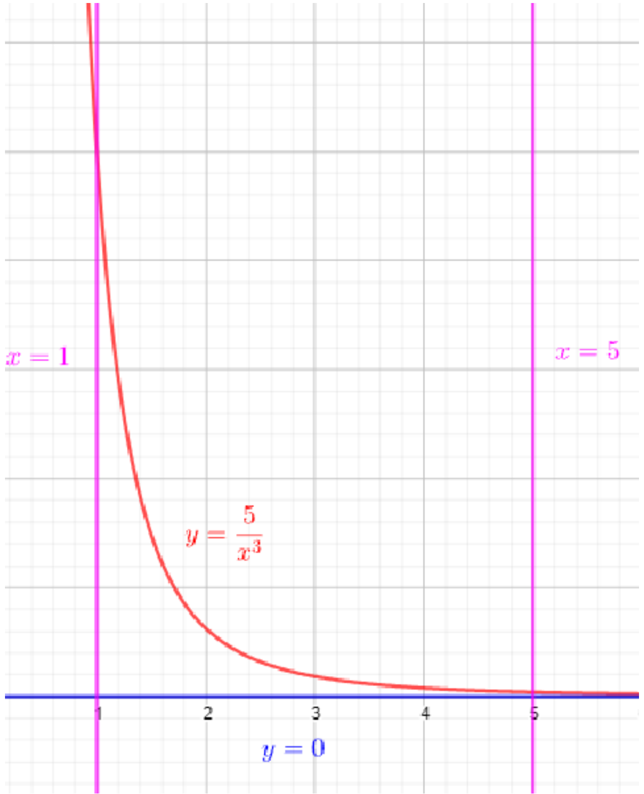


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Example 3: Find the volume obtained by revolving the region bounded by the curves

$$x = y^2 - 2y \text{ and } \qquad x = 4y - y^2$$

About the x-axis.

https://www.geogebra.org/m/f3wrypfh#material/qrar4br5

Example 3: Find the volume obtained by revolving the region bounded by the curves

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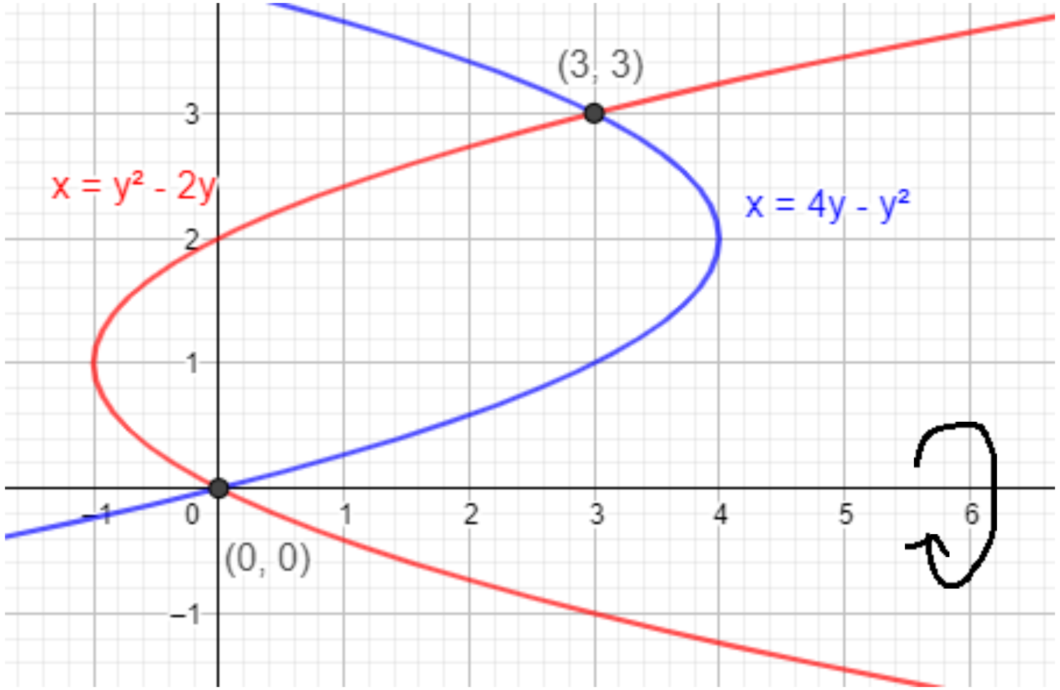
About the x-axis.

Draw the region.




https://www.geogebra.org/m/f3wrypfh#material/qrar4br5

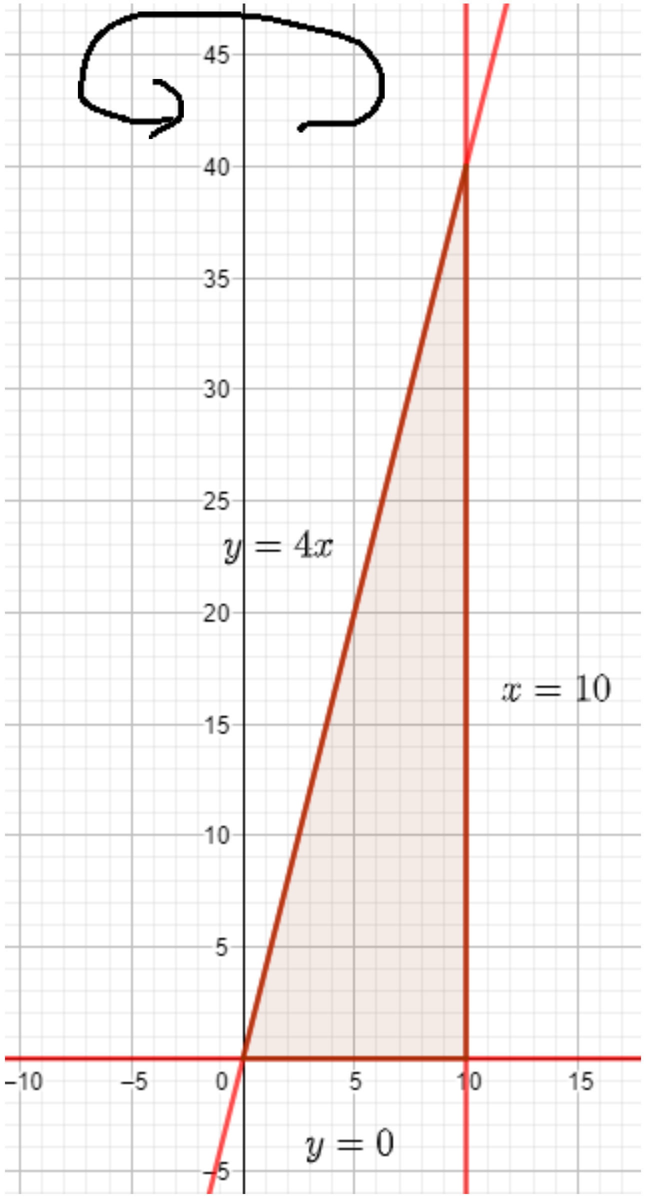
Example 6: Consider the region bounded by:

$$y = 4x\ , \qquad y = 0\ , \quad \text{and} \quad x = 10$$

Set up the integral that represents the volume of solid obtained by the rotating the region about the y-axis using

A) Disk/Washer Method
B) Shell Method

Draw the region.

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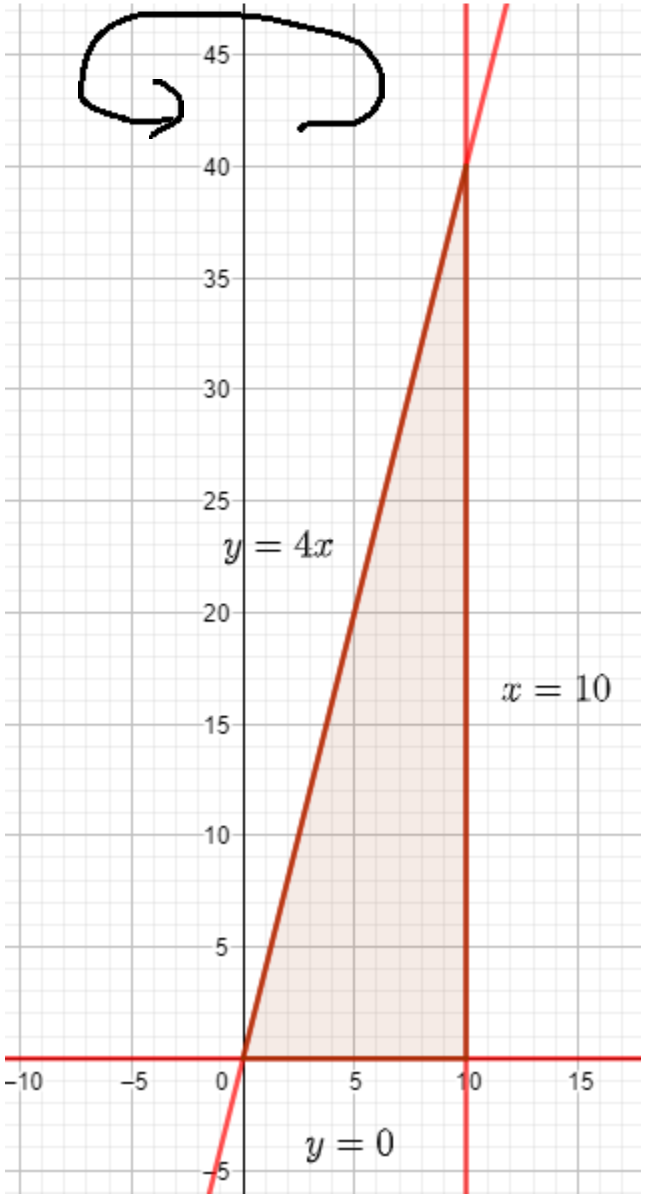


Interesting Question: Which integral is easier to compute?

A) Disk/Washer Method

$$V = \pi \int_0^{40} (10)^2 - \left(\frac{y}{4}\right)^2 \, dy$$

B) Shell Method

$$V = 2\pi \int_0^{10} x(4x)\, dx$$

https://www.geogebra.org/m/f3wrypfh#material/cb7tfq8n

# When do we apply Disk Method or Washer Method or Shell Method?

- When the region **"hugs"** the axis of rotation
  $\Rightarrow$ **Disk Method**

- When there is a **"gap"** between the region and axis of rotation
  $\Rightarrow$ **Washer Method**

- But if you find **solving for $x$ or $y$**, in either method, **is hard**
  $\Rightarrow$ **Shell Method**

| | | Axis of Rotation | |
|---|---|---|---|
| | | x-axis | y-axis |
| Method | Disk/Washer | dx | dy |
| | Shells | dy | dx |

# Formulas from Lessons 14 and 15 and 17-18 Rotation around x-axis or y-axis

**For rotation around x-axis:**

- Disk Method:

$$V = \pi \int_a^b [f(x)]^2 \, dx$$

- Washer Method:

$$V = \pi \int_a^b \left(R^2 - r^2\right) dx$$

- Shell Method:

$$V = 2\pi \int_c^d y \cdot (Right - Left) \, dy$$

**For rotation around y-axis:**

- Disk Method:

$$V = \pi \int_c^d [g(y)]^2 \, dy$$

- Washer Method:

$$V = \pi \int_c^d \left(R^2 - r^2\right) dy$$

- Shell Method:

$$V = 2\pi \int_a^b x \cdot (Top - Bottom) \, dx$$