MA 16020 - Applied Calculus II

Course Day & Time: Mon / Wed. / Fri

7:30 - 8:20 AM Section 160

8:30 - 9:20 AM Section 150

Classroom. WALC 3127

Office hrs: TBD

Lesson 1. Review of Differentiation

Helpful rules

Exponential Rules

① $x^a x^b = x^{a+b}$

② $\frac{x^a}{x^b} = x^{a-b}$

③ $(x^a)^b = x^{a \cdot b}$

④ $x^1 = x$

⑤ $x^0 = 1$

⑥ $x^{-1} = \frac{1}{x}$

Logarithmic Rules

① $\ln 1 = 0$

② $\ln(e^x) = x$

③ $e^{\ln x} = x$

④ $\ln(xy) = \ln x + \ln y$

⑤ $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

⑥ $\ln(x^m) = m \ln x$

Rational powers

① $\sqrt{x} = x^{1/2}$

② $\sqrt[3]{x} = x^{1/3}$

③ $\sqrt[q]{x^p} = x^{p/q}$

Trigonometry

	0	π/6	π/4	π/3	π/2
sin	$0 = \frac{0}{2}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2} = 1$
cos	$1 = \frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	$\frac{0}{2} = 0$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$ $\csc\theta = \frac{1}{\sin\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

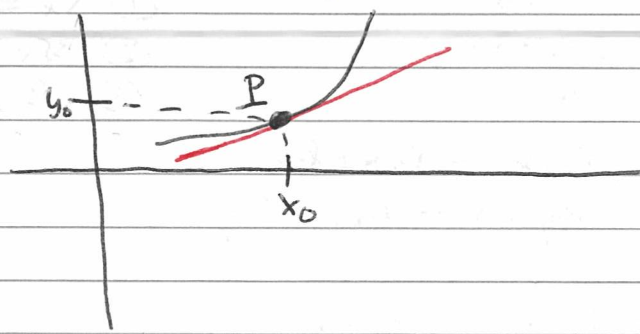
A. What is a Derivative?

Importance to <u>all</u> measurements

(sci, eng., econ., polit. sci, biology, biochemistry)

GEOM INTERP

FIND the tangent line to $\boxed{y=f(x)}$
at $P=(x_0,y_0)$.



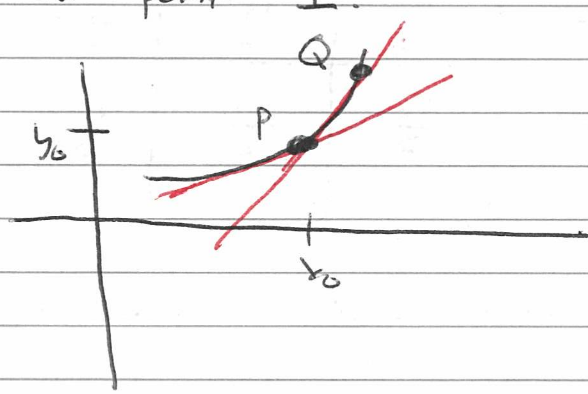
Point-Slope form

$y-y_0=m(x-x_0)$

point $y_0=f(x_0)$

Slope $m=f'(x_0)$

DEFIN $f'(x_0)$ derivative of f at x_0
is slope of tangent line to $y=f(x)$
at point P.

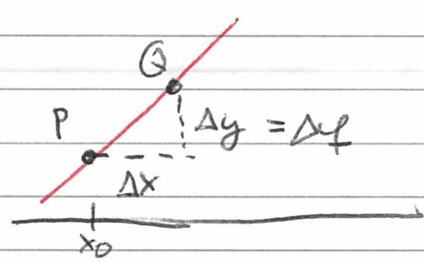


tangent line

= limit of secant lines PQ

as $Q \to P$

(P fixed)



$$\underbrace{m}_{\text{slope of tangent line}} = \lim_{\Delta x \to 0} \underbrace{\frac{\Delta f}{\Delta x}}_{\text{slope of secant line}}$$

Examples 2. Find the derivative of the following.

① $y = \dfrac{15}{3\sqrt{x^2+1}}$

Answer. $y = 15\,(x^2+1)^{-1/3}$

Use chain rule: Outside: $f(x) = 15x^{-1/3}$ inside: $g(x) = x^2+1$

$f'(x) = 15(-1/3)\,x^{-4/3}$ $g'(x) = 2x$

$= -5x^{-4/3}$

$$y' = f'(g(x))\,g'(x)$$
$$= f'(x^2+1)\cdot 2x$$
$$= -5(x^2+1)^{-4/3}\cdot 2x = \boxed{\dfrac{-10x}{(x^2+1)^{4/3}}}$$

② $y = \sin(x^3)$

Answer. Use chain rule:

Outside $f(x) = \sin x$ inside $g(x) = x^3$

$f'(x) = \cos x$ $g'(x) = 3x^2$

$$y' = f'(g(x))\cdot g'(x)$$
$$= f'(x^3)\cdot(3x^2)$$
$$= \cos(x^3)\cdot 3x^2 = \boxed{3x^2\cos(x^3)}$$

③ $W(z) = \dfrac{3z+9}{2-z}$ Answer: Use quotient rule

$g(z) = 2-z$

$f(z) = 3z+9$ $\Rightarrow$ $\boxed{W(z) = \dfrac{f(z)}{g(z)}}$

$$W'(z) = \frac{2-z\ (3)\ (-)\ (3z+9)(-1)}{(2-z)^2}$$

$$= \frac{6-3z\ (-)\ (-3z-9)}{(2-z)^2}$$

$$= \frac{(-3z+3z)+(6+9)}{(2-z)^2}$$

$$= \boxed{\frac{15}{(2-z)^2}}$$

$$\boxed{f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}}$$

$(= m)$

Difference quotient

here: $\Delta f = f(x_0 + \Delta x) - f(x_0)$

Rules of Differentiation

Constant rule: $\frac{d}{dx}(c) = 0$ where c is constant

Power rule: $\frac{d}{dx}(x^n) = nx^{n-1}$, n is real number

Constant Multiple rule: $\frac{d}{dx}[c\,f(x)] = c\frac{d}{dx}[f(x)]$, c is constant

Sum / Difference Rule: $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

Examples. Find the derivative of:

① $f(x) = x^5$

Answer: $\boxed{f'(x) = 5x^4}$

② $f(x) = 1/x$

Answer. $f(x) = x^{-1}$

$$\boxed{f'(x) = -x^{-2} = \frac{-1}{x^2}}$$

③ $f(x) = \frac{3}{x^4} - 2x^2 + 6x - 7$

Answer. $f(x) = 3x^{-4} - 2x^2 + 6x - 7$

$f'(x) = 3(-4)x^{-5} - 2(2)x + 6$

$= -12x^{-5} + -4x + 6$

$= \boxed{\frac{-12}{x^5} - 4x + 6}$

(4) $f(x) = \dfrac{x^{2.5} - 2x^{-3}}{x}$

Answer $f(x) = \dfrac{x^{2.5}}{x} - \dfrac{2x^{-3}}{x} = x^{2.5-1} - 2x^{-3-1}$

$= x^{1.5} - 2x^{-4}$

$f'(x) = 1.5x^{0.5} - 2(-4)x^{-5}$

$= \boxed{1.5x^{0.5} + 8x^{-5}}$

Other Differentiation Rules

$\dfrac{d}{dx}(\sin x) = \cos x$ $\qquad$ $\dfrac{d}{dx}(\cos x) = -\sin x$

$\dfrac{d}{dx}(\tan x) = \sec^2 x$ $\qquad$ $\dfrac{d}{dx}(\cot x) = -\csc^2 x$

$\dfrac{d}{dx}(\sec x) = \tan x \sec x$ $\qquad$ $\dfrac{d}{dx}(\csc x) = -\csc x \cot x$

$\dfrac{d}{dx}(e^x) = e^x$ $\qquad$ $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$

Product rule. $\dfrac{d}{dx}(u(x)\,v(x)) = u'(x)\,v(x) + u(x)\,v'(x)$

Quotient rule : $\dfrac{d}{dx}\left[\dfrac{u(x)}{v(x)}\right] = \dfrac{u'(x)\,v(x) - u(x)\,v'(x)}{v^2(x)}$

Chain Rule. $h(x) = f(g(x))$.

$h'(x) = f'(g(x)) \cdot g'(x)$