Formula Sheet - MA 16020

First-Order Linear Differential Equations
Any solution $y = f(x)$ of a first-order linear differential equation in standard form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

must satisfy $y \cdot u(x) = \int u(x)Q(x)dx$ where $u(x) = e^{\int P(x)dx}$.

Geometric Series:
The geometric series $\sum_{n=0}^{\infty} ar^n$ with common ratio r converges if $|r| < 1$ with the sum

$$S = \sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

Power Series/Maclaurin Series:

$$\begin{aligned}
\frac{1}{1-x} &= \sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + \ldots \qquad |x| < 1 \\
e^x &= \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots \\
\sin x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots \\
\cos x &= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots \\
\ln(1+x) &= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots \qquad |x| < 1
\end{aligned}$$

Second Derivative Test
Given the critical point (a, b), such that $f_x(a, b) = 0$ and $f_y(a, b) = 0$, and let

$$D(a, b) = f_{xx}(a, b) f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

- If $D > 0$ and $f_{xx}(a, b) > 0$ then $f(a, b)$ is a relative minimum.
- If $D > 0$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a relative maximum.
- If $D < 0$, then $f(a, b)$ is a saddle point.
- If $D = 0$, the test is inconclusive.